

THE MODIFICATION OF MARKOWITZ PORTFOLIOS MADE FOR STOCKS FROM THE WARSAW STOCK EXCHANGE

Рассматривается метод расчета альтернативных ограничений для модели Марковича. Три измененных и одна стандартная модели Марковича дают четыре структуры пакета ценных бумаг с разными коэффициентами предотвращения крайнего риска. Структура инвестиций каждого из пакетов ценных бумаг зависит от коэффициента предотвращения крайнего риска, параметров вспомогательных функций для выбранных инвестиций и соотношения акций пакета и акций на фондовом рынке в выбранные периоды и для определенного числа акций. Источником данных для анализа послужили Варшавская фондовая биржа и период планирования с декабря 2000 по декабрь 2004 г.

This paper presents method of construction alternative constraints for the Markowitz model. Three models with modification and one standard Markowitz problem gives four structures of stock portfolios with different coefficients of extreme risk aversion. Each portfolio investment space structure depends on coefficient of extreme risk aversion, shape of utility function for selected investment and ratio of shares in portfolio to shares in stock exchange in selected periods and amount of shares. Date sets for analysis come from WSE and from a 49 months planning horizon: December 2000 to December 2004.

1. Introduction

This paper presents state of knowledge of finding optimal solution to the quadratic programming Markowitz model with modifications of linear constraints. It presents investment space structure for the four stock exchange portfolios. Each portfolio has strong relation between utility function drafted according to computed extreme risk aversion coefficient and proportion of shares in portfolio to shares in stock exchange in selected periods. It is a huge problem for adaptations of classical model to specific problems of new markets such as Warsaw Stock Exchange.

2. Data used for computations

For construction of Markowitz portfolio problem data from WSE form forth nine months period – December 2000 to December 2004. The historic statistical analysis of time series from selected periods have been placed in database constructed from economic web site in biznes.interia.pl. Data were standardized in each month. For this analysis two type of figures have been used: share of investment in turnover of stock exchange in selected month

and mean price change of investment in selected month.

3. Structure of investment portfolio and the Markowitz problem with modification

Let $i = 1, 2, \dots, n$ be the index and n – total number of investments available in the stock exchange or taken into consideration as a potential investment direction.

Let ϖ_i be the capital assigned to investment i .

$$\text{If } \sum_{i=1}^n \varpi_i = \varpi \text{ and } \varpi > 0, \quad (1.0)$$

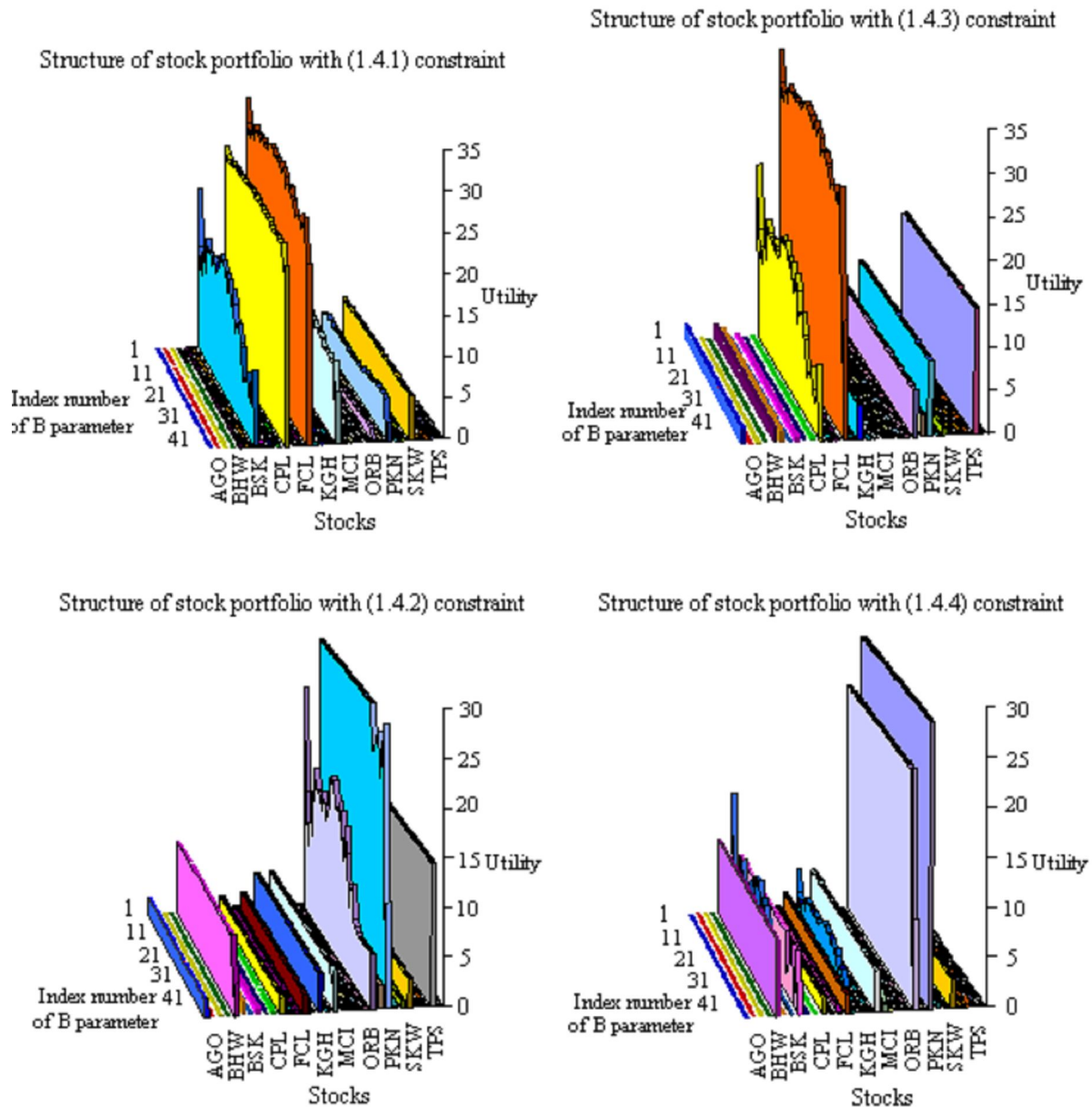
$$\text{then } x_i = \frac{\varpi_i}{\varpi} (i = 1, 2, \dots, n), \quad (1.1)$$

is the share of i th investment of the portfolio.

Variable x_i is defined to be a weight of investment i in the portfolio. The weights x_i create an n -element vector of shares, which is defined as a structure of investment portfolio. All feasible vectors of shares in stock portfolios create a class, which is defined as the investment space.

AGO	BPH	CMR	FCL	KRB	NET	PKN	SPT
AMC	BRE	CPL	FSC	KTY	ORB	PLC	STX
BDX	BSK	DBC	JLF	MCI	PEO	SFT	TPS
BHW	BZW	ELE	KGH	MIL	PKM	SKW	ZWC

Table 1. The structure of the investment portfolio



The Markowitz Problem is known as a standard problem of selection the optimal portfolio. The modified mathematical model is presented below:

For all $k = 1, \dots, m$ maximize

$$Z(k) = \sum_{i=1}^n x_i E(R_i) - \lambda \sum_{i=1}^n \sum_{j=1}^n x_i x_j \text{cov}(R_i, R_j), \quad (1.2)$$

subject to

$$\sum_{i=1}^n x_i = \beta_k \quad (1.3)$$

$$\text{and} \quad x_i \geq 0, \quad i = 1, \dots, n \quad (1.4.1)$$

$$\text{or} \quad x_i^{\min} \leq x_i \leq x_i^{\max}, \quad i = 1, \dots, n \quad (1.4.2)$$

$$\text{or} \quad x_i \geq x_i^{\min}, i = 1, \dots, n \quad (1.4.3)$$

$$\text{or} \quad 0 \leq x_i \leq x_i^{\max}, i = 1, \dots, n \quad (1.4.4)$$

where: x_i share of i th investment of the portfolio,

$x_i^{\min}$ denotes the least average share of selected investment in turnover of stock exchange,

$x_i^{\max}$ denotes the largest average share of selected stock in turnover of stock exchange,

λ parameter lambda, describes the level of risk aversion for individual investor,

β_k parameter Beta, proportion (ratio) of shares in portfolio to shares in stock market in selected periods,

$\text{cov}(R_i, R_j)$ matrix of covariance of all mean price change of investment in the portfolio,

$E(R_i)$ expected value of return on i th investment.

4. Computational results

The risk aversion coefficients were calculated using Excel in Solver module attributed zero value for the objective function. F

zero value for the objective function. For the standard Markowitz problem the computations produce more than one solution. Below four investment space structures of stock portfolios are presented according to the classical model and the models with alternative constraints:

5. Conclusions

The portfolios investment space structures show various dependences characteristics. Each structure depends on the coefficient of extreme risk aversion, the shape of the utility function for selected investment and the ratio of shares in portfolio to shares in stock exchange in selected periods and amount of shares.

REFERENCES

1. SAWIK B., 2004, The Calculation of utility and risk aversion for investment portfolios of the Warsaw Stock Exchange, Kraków, UWND-AGH, ISBN 83-89388-63-4.
2. SAWIK B., 2003, New approach to investment portfolios, Kraków, STN, ISBN 83-918282-1-7.
3. SAWIK B., 2001, Risk aversion control in investment portfolios, WZ-AGH, Kraków, ISBN 83-914035-8-0.