MICHAŁ MISCIORAK, GRZEGORZ JANIK
Agriculture University of Wroclaw, Poland

APPLICATION OF FRACTAL DIMENSION THEORY FOR THE ESTIMATION OF SPATIAL VARIABILITY OF SOIL PARAMETERS

This work aims to estimate stability of variograms used for estimation of spatial variability of moisture. The analysis is based on soil moisture measurement taken at the depth of 10 cm below the ground level. The research took place in Sucha Rzeczka town at warmińsko-mazurskie voivodeship, on the greenland. The research allowed to construct moisture variograms for $N_{max} = 40$ and to estimate fractal dimension. According to the measurements and analysis it has been ascertained that the stability of variograms is satisfying when they are constructed for at least 20 compared pairs of measurement points. The high value of fractal dimension $D = 1{,}8$ proves that the randomness of moisture in the examined area is high. Fractal dimension value stabilizes when it is estimated on the basis of variograms constructed for at least 30 compared points.

Цель работы – оценка стабильности вариограмм, используемых для определения пространственной изменчивости влагоемкости. Анализ базируется на измерении влагоемкости почвы, взятой с глубины 10 см ниже уровня поверхности. Исследования проводились в городе Sucha Rzeczka Варминско-Мазурского воеводства, на лугопастбищных угодьях. Исследования позволили построить влажностные вариограммы для $N_{max} = 40$ и оценить фрактальные размеры. На основе измерений и анализа было установлено, что стабильность вариограмм является удовлетворительной в случае, если они основываются на 20 сравниваемых пар точек измерения. Высокое значение фрактального размера $D = 1{,}8$ подтверждает, что влагоемкость на изучаемой территории высока. Значение фрактального размера стабилизируется в случае, если оно оценивается на основе вариограмм, построенных для 30 точек сравнения.

DESCRIPTION OF THE METHOD AND OF THE EXPERIMENT

The experiment was conducted in Sucha Rzeczka town in the north – east of Poland, on an experimental field constituting a grassland. Soil moisture measurements were carried out at the depth of 10 cm below the ground level. The distance, between measurement points situated along the 60 m long section, was 1 m or 0,2 m. The estimation of spatial variability of soil moisture is based on the analysis of variograms, which were defined as a function of moisture variance depending on distance between compared points.

The variogram and its particular points were calculated according to the formula 1[WALCZAK et al. 1994; BRANDYK et al.1995; USOWICZ 1999; USOWICZ et al. 2000; JANIK 2005]:

$$\gamma(d) = \frac{1}{2N(d)} \sum_{i=1}^{N(d)} \left(w_i - w_{i+d}\right)^2 , \qquad (1)$$

where: $\gamma(d)$ – variance; $N(d)$ – number of the compered pairs of moisture values at distance d; w_i – soil moisture in point i; w_{i+d} – soil moisture in point $i + d$.

OBTAINED RESULTS

Figure 1 illustrates the results of moisture measurements.

The variogram was constructed using dependence 1. Available measurement data allowed to calculate the value of function $\gamma(d)$ for

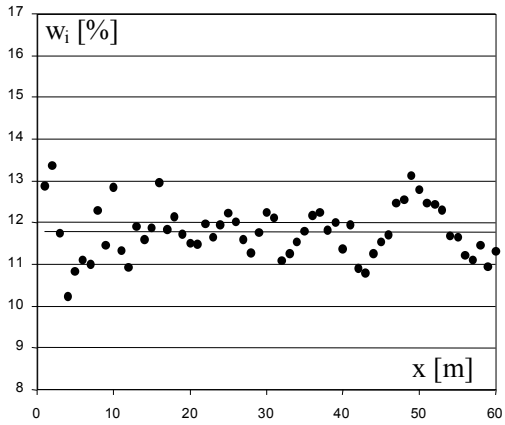

Fig.1. Moisture measurement at the depth of 10 cm below the ground level lengthways the length of 60 m

the distance between measurement points d = 0,2÷21,0 m. Calculations were made for the number of compared pairs of points N = 5÷ 40. The variograms are presented at Figure 2.

Next, fractal dimension D, whose value amounted to 1.8, was calculated. The analysis of variograms shows that concentration of calculated values $\gamma(d)$ around the function (estimated for N = 40) grows along with the increment of N number. This observation was brought under quantitative assessment by introduction of conformity criterion of value $\gamma(d)$ to function $\gamma_m(d)$, estimated on the basis of formula 2:

$$B_N = \frac{1}{M}\sum_{j=1}^{M}\left|\gamma(d_N) - \gamma_m(d_{N\max})\right|, \qquad (2)$$

where: B_N –value $\gamma(d)$ conformity criterion and $\gamma_m(d_{N\max})$ estimated on the basis of dependence 4 for varying N number; $\gamma(d_{N\max})$ – variance calculated on the basis of dependence 1; $\gamma_m(d)$ – variance calculated on the basis of dependence 2 for $N_{\max}$ = 40; M – number of points in which the values of $\gamma(d)$ and $\gamma_m(d_{N\max})$, were compared (M = 26)

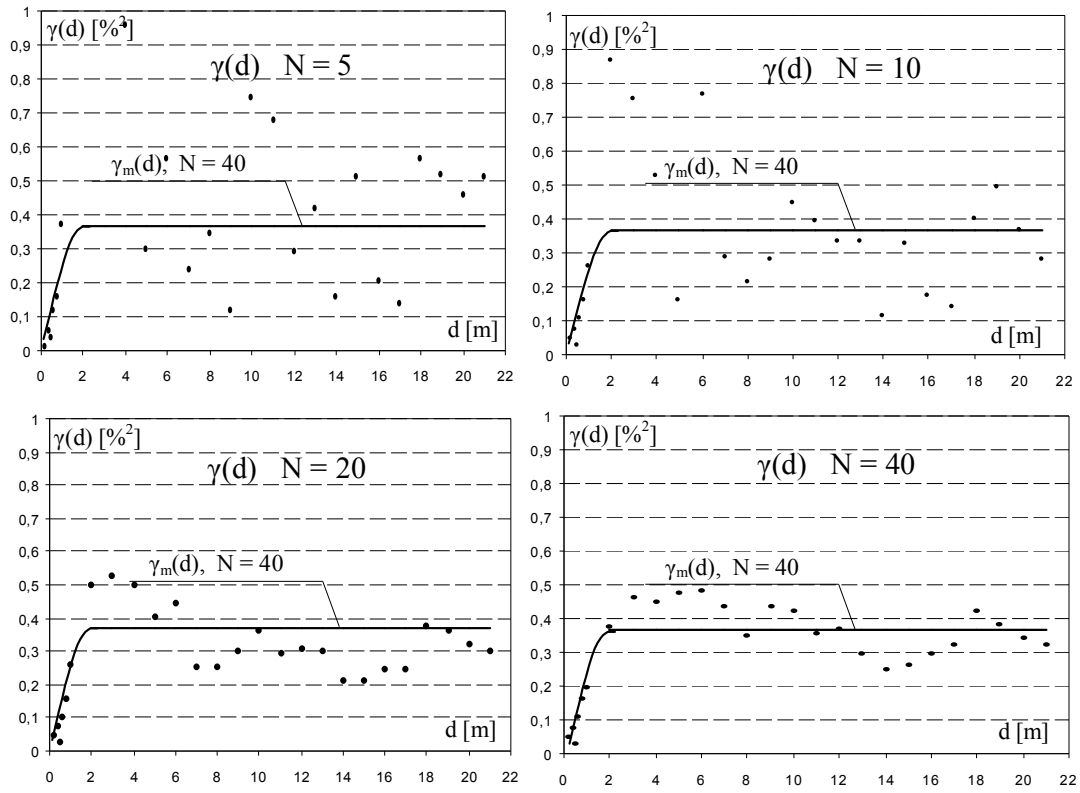

Fig.2. Moisture variograms constructed on the basis of varying number of compared pairs. N – number of compared pairs, d – distance between compared points, $\gamma(d)$ – variance estimated according to formula 1, $\gamma_m(d)$ – approximating function of variance and distance dependence

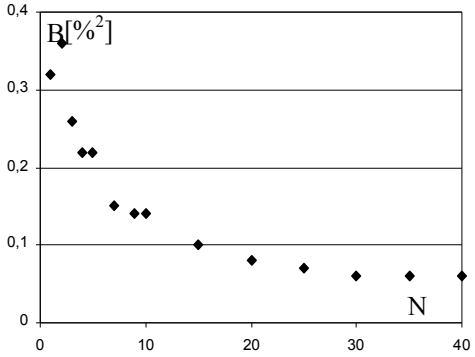

Fig.3. Accordance of variograms constructed for varying N number and variogram $\gamma_m(d)$ for $N = 40$

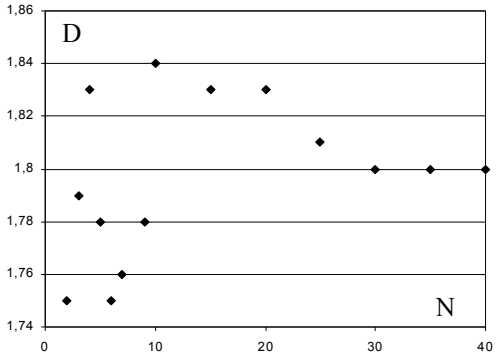

Fig.4. Fractal dimension stability

Figure 3 shows that when variograms are constructed on the basis of at least 20 compared pairs of points, then their conformity with real variogram ($n = 40$) is satisfactory. Variograms are stabile for $N > 20$.

Figure 4 shows the variability of fractal dimension D estimated on the basis of variograms constructed for N value from 2 to 40. In this case the value of D stabilises for $N > 30$.

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