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SMALL SPEED ASYMPTOTIC STABILITY STUDY OF A SPEED VECTOR CONTROL SYSTEM FOR AN INDUCTION MOTOR THAT CONTAINS IN ITS LOOP A GOPINATH OBSERVER

In this paper we analyze the asymptotic stability of a vector control system for an induction motor with short-circuited rotor that contains in its loop a Gopinath observer. The studied control system is based on the direct rotor flux orientation method (DFOC) and the stability study is based upon the linearization theorem around the equilibrium points of the control system, emphasizing the estimated variation domain of the rotor resistance for which the control system remains asymptotically stable when the prescribed speed of the control system is close to zero. The stability study is made in both the continual and discrete cases. The mathematical model of the vector regulating system is made using a value $d\lambda_e$–$q\lambda_e$ linked to stator current. In order to mathematically describe the DFOC control system we will consider the following hypotheses: the static frequency converter (CSF) is assumed to contain a tension inverter; the static frequency converter is considered ideal so that the vector of the command measures is considered to be the entry vector of the induction motor; the dynamic measure transducers are considered ideal; the system and axis transformation blocks are considered dynamically ideal; the mathematical model of the vector control system will be written in an $d\lambda_e$–$q\lambda_e$ axis reference bounded to the stator current.

В статье анализируется асимптотическая стабильность векторной системы управления для асинхронного двигателя с короткозамкнутым ротором, которая содержит наблюдатель Гопина. Рассмотренная система управления базируется на прямом методе ориентации потока ротора (DFOC), а исследование стабильности основано на теореме линеаризации возле точек равновесия системы управления. При этом акцент сделан на предполагаемой области изменения сопротивления ротора, для которой система управления остается асимптотически устойчивой, когда предписанная скорость системы управления близка к нулю. Исследование стабильности проводилось в непрерывных и в дискретных случаях. Математическая модель векторной системы регулирования создана с использованием значений $d\lambda_e$–$q\lambda_e$, связанных с потоком статора. Для математического описания системы управления DFOC, исходим из следующих предположений: статический преобразователь частоты (CSF), как предполагается, содержит инвертор напряженности; статический преобразователь частоты считают идеальным так, чтобы вектор управления был вектором входа асинхронного двигателя; динамические преобразователи измеренных значений считаются идеальными; система и блоки преобразования координат считаются динамически идеальными; математическая модель векторной системы управления будет создана в осях $d\lambda_e$–$q\lambda_e$, связанных с потоком статора.

1. Introduction. This paper approaches a difficult problem within vector driving systems for induction motors namely the asymptotic stability study in a Lyapunov manner. The difficulty is brought by the mathematical model of the nonlinear analyzed system that makes the Lyapunov stability analysis methods difficult to apply. The novelty of the paper consists in obtaining the method and the form of the linerized mathematical model on which the analysis of the asymptotic stability is made. The shape and structure of the mathematical model depends on the actual components within the analyzed control system and the way in which the state values are selected. In this paper we present the analysis of the asymptotic stability of a vector control system for an induction motor with a short-circuited rotor that contains in its loop a Gopinath observer. The analyzed control system is orientated according to the estimated rotor flux by the Gopinath observer.

We have realized the stability analysis for low speeds, emphasizing the influence of the iden-

tified rotor resistance within the stability of the control system. We have obtained the variation range of the identified rotor resistance for which the control system remains asymptotically stable.

2. The Mathematical Description of the Vector Control System. In order to mathematically describe the DFOC control system we will consider the following hypotheses:

The static frequency converter (CSF) is assumed to contain a tension inverter.

The static frequency converter is considered ideal so that the vector of the command measures is considered to be the entry vector of the induction motor.

The dynamic measure transducers are considered ideal.

The system and axis transformation blocks are considered dynamically ideal.

The mathematical model of the vector control system will be written in an $d\lambda_e$–$q\lambda_e$ axis reference bounded to the stator current.

Based on these hypotheses the block diagram of the direct vector control system that contains a Gopinath type rotor flux estimator in its loop is presented within fig.1 and the DFOC field orientation block within fig.2.

Some of the equations that define the vector control system are given by the elements which compose the field orientation block and consist of:

- stator tensions decoupling block (C_1U_s);
- couple *PI* regulator (PI_M_e) defined by the K_M proportionality constant and the T_M integration time;
- *PI* flux regulator (PI_ψ) defined by the K_ψ proportionality constant and the T_ψ integration time;
- mechanical angular speed *PI* regulator (PI_W) defined by the K_ω proportionality constant and the T_ω integration time;
- current *PI* regulator (PI_I) defined by the K_i proportionality constant and the T_i integration time;
- Flux analyzer (*AF*).

The other equations that compose the mathematical model of the speed control system are those given by the relations that define the stator currents – rotor fluxes mathematical

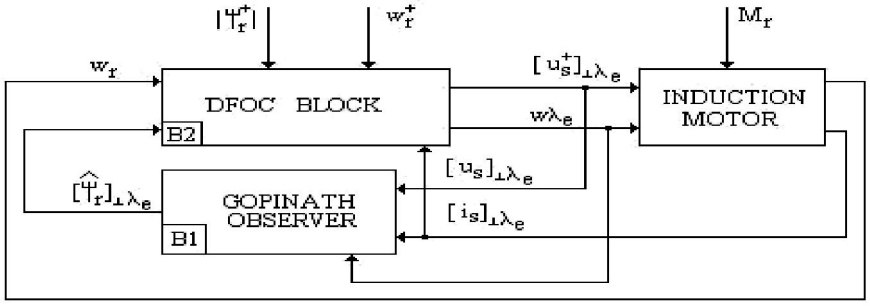


Fig.1. Direct Vector Speed Control System

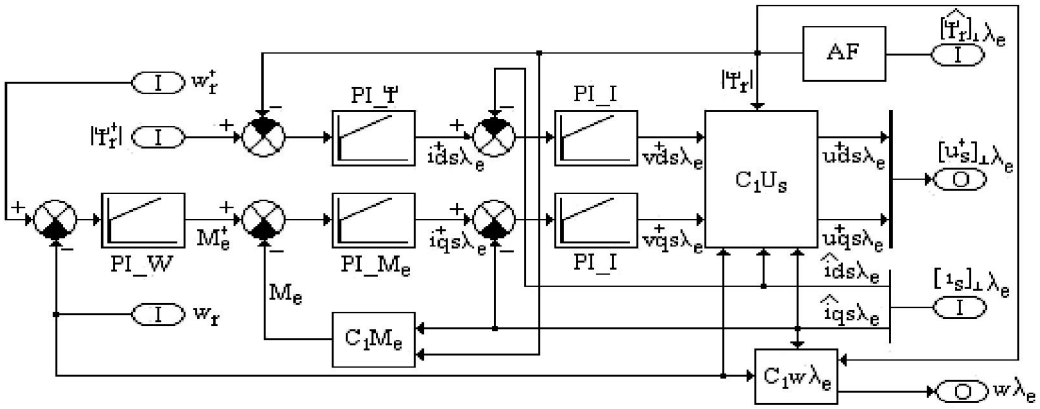


Fig.2. Internal structure of the field orientation bloc (DFOC)

model of the induction motor as well as the equations that define the mathematical model of the observer for the Gopinath type rotor flux.

All these expressions can be put together as a 12 differential equations system with 12 unknown values. In order to offer a coherent presentation of this differential equations system, we have used the following notations:

The state vector of the control system will be

$$x = [xi]_{i=\overline{1,12}}$$

where $x_1 = i_{ds\lambda_e}$; $x_2 = i_{qs\lambda_e}$; $x_3 = \psi_{dr\lambda_e}$; $x_4 = \psi_{qr\lambda_e}$; $x_5 = \omega_r$; $x_{11} = \hat{\psi}_{dr\lambda_e}$; $x_{12} = \hat{\psi}_{qr\lambda_e}$

The input vector of the of the control system will be

$$u = [u_1 u_2 u_3]^T$$

where $u_1 = \psi_r^*$; $u_2 = \omega_r^*$; $u_3 = M_r$.

Under these circumstances the 12 differential equations system that define the mathematical model of the vector regulating system can be written as follows

$$\frac{d}{dt}x(t) = f(x(t), u(t)), \qquad (1)$$

where $f(x,u) = [f_i(x,u)]_{i=\overline{1,12}}$, and the $f_i = f_i(x,u)$ functions are:

$$f_1(x,u) = a_{11}x_1 + \left(z_p x_5 + a_{31}^* \frac{x_2}{\sqrt{x_{11}^2 + x_{12}^2}}\right) \times$$
$$\times x_2 + a_{13}x_3 + a_{14}z_p x_5 x_4 + b_{11}u_{ds\lambda_e};$$

$$f_2(x,u) = -\left(z_p x_5 + a_{31}^* \frac{x_2}{\sqrt{x_{11}^2 + x_{12}^2}}\right) \times$$
$$\times x_1 + a_{11}x_2 - a_{14}z_p x_5 x_3 + a_{13}x_4 + b_{11}u_{qs\lambda_e}$$

$$f_3(x,u) = a_{31}x_1 + a_{33}x_3 + a_{33}^* \frac{x_2}{\sqrt{x_{11}^2 + x_{12}^2}} x_4;$$

$$f_4(x,u) = a_{31}x_2 - a_{31}^* \frac{x_2}{\sqrt{x_{11}^2 + x_{12}^2}} x_3 + a_{33}x_4;$$

$$f_5(x,u) = K_{m1}[x_3 x_2 - x_4 x_1] - K_{m2}x_5 - K_{m3}u_3;$$

$$f_6(x,u) = u_1 - \sqrt{x_{11}^2 + x_{12}^2};$$

$$f_7(x,u) = \frac{K_\omega}{T_\omega}x_8 + K_\omega(u_2 - x_5) -$$
$$- K_a x_2 \sqrt{x_{11}^2 + x_{12}^2};$$
$$f_8(x,u) = u_2 - x_5;$$

$$f_9(x,u) = \frac{K_\psi}{T_\psi}x_6 + K_\psi\left(u_1 - \sqrt{x_{11}^2 + x_{12}^2}\right) - x_1;$$

$$f_{10}(x,u) = \frac{K_M}{T_M}x_7 + K_M \times$$
$$\times\left(\frac{K_\omega}{T_\omega}x_8 + K_\omega(u_2 - x_5) - K_a x_2 \sqrt{x_{11}^2 + x_{12}^2}\right) - x_2;$$

$$f_{11}(x,u) = b_1 x_{11} + b_2 x_{12} - g_a b_{11}^* u_{ds\lambda_e} +$$
$$+ g_b b_{11}^* u_{qs\lambda_e} + b_3 x_1 + b_4 x_2 +$$
$$+ g_a f_1(x,u) - g_b f_2(x,u);$$

$$f_{12}(x,u) = -b_2 x_{11} + b_1 x_{12} - g_a b_{11}^* u_{ds\lambda_e} -$$
$$+ g_b b_{11}^* u_{qs\lambda_e} + b_4 x_1 + b_3 x_2 +$$
$$+ g_b f_1(x,u) + g_a f_2(x,u).$$

Where:

$$a_{11} = \left(\frac{1}{T_s\sigma} + \frac{1-\sigma}{T_r\sigma}\right); \quad a_{13} = \frac{L_m}{L_s L_r T_r \sigma};$$

$$a_{14} = \frac{L_m}{L_s L_r \sigma}; \quad a_{31} = \frac{L_m}{T_r}; \quad a_{33} = -\frac{1}{T_r};$$

$$b_{11} = \frac{1}{L_s\sigma}; \quad T_s = \frac{L_s}{R_s}; \quad T_r = \frac{L_r}{R_r}; \quad \sigma = 1 - \frac{L_m^2}{L_s L_r};$$

$$K_{m1} = \frac{3}{2}\frac{z_p}{J}\frac{L_m}{L_r}; \quad K_{m2} = \frac{F}{J}; \quad K_{m3} = \frac{1}{J};$$

$$K_a = \frac{3}{2}z_p \frac{L_m^*}{L_r^*};$$

$$a_{11}^* = \left(\frac{1}{T_s^*\sigma^*} + \frac{1-\sigma^*}{T_r^*\sigma}\right); \quad a_{13}^* = \frac{L_m^*}{L_s^* L_r^* T_r^* \sigma};$$

$$a_{14}^* = \frac{L_m^*}{L_s^* L_r^* \sigma^*}; \quad a_{31}^* = \frac{L_m^*}{T_r^*}; \quad a_{33}^* = -\frac{1}{T_r^*};$$

$$b_{11}^* = \frac{1}{L_s^*\sigma^*}; \quad T_s^* = \frac{L_s^*}{R_s^*}; \quad T_r^* = \frac{L_r^*}{R_r^*};$$

$$\sigma^* = 1 - \frac{L_m^{2\,*}}{L_s^* L_r^*};$$

$$\alpha = k\sqrt{z_p^2 x_5^2 + (a_{33}^*)^2};$$

$$g_a = -\frac{1}{a_{14}^*}\frac{1+\alpha a_{33}^*}{z_p^2 x_5^2 + (a_{33}^*)^2};$$

$$g_b = -\frac{1}{a_{14}^*}\frac{\alpha z_p x_5}{z_p^2 x_5^2 + (a_{33}^*)^2};$$

$$b_1 = a_{33}^* - g_a a_{13}^* - g_b a_{14}^* z_p x_5 +$$

$$+ z_p x_5 + a_{31}^* \frac{x_2}{\sqrt{x_{11}^2 - x_{12}^2}};$$

$$b_2 = g_b a_{13}^* - (1 + g_a a_{14}^*) z_p x_5;$$

$$b_3 = a_{31}^* - g_a a_{11}^* - g_b \times$$

$$\times \left(z_p x_5 + a_{31}^* \frac{x_2}{\sqrt{x_{11}^2 + x_{12}^2}} \right);$$

$$b_4 = g_b a_{11}^* - g_a \left(z_p x_5 + a_{31}^* \frac{x_2}{\sqrt{x_{11}^2 + x_{12}^2}} \right);$$

$$u_{ds\lambda e} = \frac{1}{b_{11}^*}(b_{11}^* v_{ds\lambda e}^* - h_1);$$

$$u_{qs\lambda e} = \frac{1}{b_{11}^*}(b_{11}^* v_{qs\lambda e}^* - h_2);$$

$$v_{ds\lambda e} = \frac{K_i}{T_i} x_9 + K_i f_9(x,u);$$

$$h_1 = a_{13}^* \sqrt{x_{11}^2 + x_{12}^2} + a_{31}^* \frac{x_2^2}{\sqrt{x_{11}^2 + x_{12}^2}} + z_p x_5 x_2;$$

$$h_2 = a_{14}^* z_p x_5 \sqrt{x_{11}^2 + x_{12}^2} +$$

$$+ a_{31}^* \frac{x_1 x_2}{\sqrt{x_{11}^2 + x_{12}^2}} + z_p x_5 x_1.$$

Where L_s, Lr, L_m are the stator, rotor and mutual inductance, R_s, R_r are the stator and rotor resistance and σ is the mutual coefficient of leakage.

In the relations above we noted with «*» the identified measures of the induction motor and k is the proportionality coefficient Gopinath.

Under these circumstances the mathematical model of the speed vector control system is fully determined as being defined by the non-linear differential equations system given by (1) whose initial condition is $x(0)=0$.

3. The Asymptotic Stability Study of the Control System. In order to realize the analysis of the asymptotic stability we will consider an induction motor that has the following characteristics:

- electrical parameters

$$R_s = 0{,}371[\Omega];\ R_r = 0{,}415[\Omega];$$
$$L_s = 0{,}08694[H];\ L_r = 0{,}08762[H];$$
$$L_m = 0{,}08422[H].$$

- mechanical parameters

$$z_p = 2;\ J = 0{,}15[kg \cdot m^2];$$
$$F = 0{,}005\left[\frac{N \cdot m \cdot s}{rad}\right].$$

On the other hand, following the automated regulators tuning within the speed control system we obtain the following constants:

$$K_\psi = 501{,}3834;\ T_\psi = \frac{K_\psi}{2374{,}7};$$

$$K_i = 5{,}9881;\ T_i = \frac{K_i}{754{,}4176};$$

$$K_M = 10{,}1988;\ T_M = \frac{K_M}{1020};$$

$$K_\omega = 10;\ T_\omega = \frac{K_\omega}{350}.$$

Under these circumstances by imposing the entry vector of the control system to be of the following type

$$u_1 = \psi_r^* = 0{,}69[Wb];$$
$$u_{2m} = w_{rm}^* = n_m \frac{\pi}{30}\left[\frac{rad}{s}\right]; \qquad (2)$$
$$u_3 = M_r = M_N = 93{,}269[N \cdot m],$$

where $n_m = 100m[rpm]$ with $m = \overline{0{,}15}$ and the proportionality coefficient between the self values of the motor and those of the observer being $k = 0{,}3$ by solving the non-linear system $f(x,u) = 0$ using the Newton method having as the start point the vector:

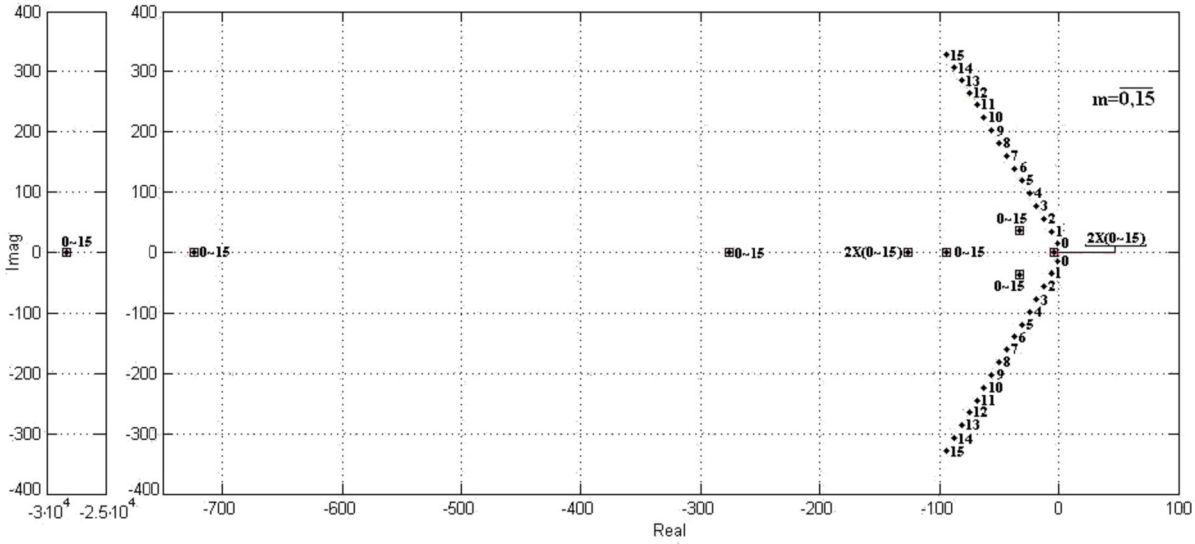


Fig.3. The self values of the matrix A_L; M_r=M_N

$x^* = [0 \quad 0 \quad u_1 \quad 0 \quad u_{2m} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad u_1 \quad 0]$ we obtain an equilibrium point $x_m = b_m$ where: $b_m = [b_{mi}]_{i=\overline{1,12}}$.

From those stated above by the linearization of the system (1) around the equilibrium point $x_m = b_m$ obtained for an $u^* = [u_1 u_{2m} u_3]^T$ entry vector defined by (2) we obtain:

$$\Delta\dot{x}(t) = A_L \Delta x(t) + B_L \Delta u(t) \qquad (3)$$

where A_L, B_L matrixes are:

$$A_L = \left[\frac{\partial f_i}{\partial x_j}(b_m, u^*)\right]_{i=\overline{1,12};\, j=\overline{1,12}};$$

$$B_L = \left[\frac{\partial f_i}{\partial u_k}(b_m, u^*)\right]_{i=\overline{1,12};\, k=\overline{1,3}}.$$

Next, in order to study the asymptotic stability of the equilibrium points x_m=b_m we will analyze the self values of the A_L matrix so that if they have a strictly negative real part the x_m=b_m equilibrium point is asymptotically stable for the linerized system (3). Under these circumstances according to the linearization theorem* in a vicinity of the equilibrium point x_m=b_m the non-linear system (1) is asymptotically stable.

As the self values of the A_L matrix are presented within fig.3, it results that the equilibrium points x_m=b_m of the linerized system (3) are asymptotically stable and according to the linearization theorem the equilibrium points x_m=b_m are asymptotically stable in certain vicinity for the non-linear system (1).

Also in fig.3 for a more in depth study of the asymptotic stability of the (1) system we emphasized the self values of the control system that does not contain a Gopinath observer.

From those presented above one may notice that the self values consist of the self values of the control system that does not contain a Gopinath observer as well as the self values of the Gopinath observer. The only self values that modify when a new entry vector is imposed are the self values of the Gopinath observer.

One may notice that inclusion of the Gopinath observer does not alter the self values of the control system that does not contain it.

Thus, one can conclude that we can study the asymptotic stability of a control system that contains in its loop a Gopinath observer by independently studying the asymptotic stability of the control system that does not contain a Gopinath observer and asymptotic stability of the Gopinath observer.

For a more detailed study of the asymptotic stability, next we will present the A_L matrix self values under identical testing condi-

* *Voicu M.* Tehnici de analiza a stabilitatii sistemelor automate. Editura Tehnica. Bucuresti, Romania, 1986.

tions with those above with the difference that $u_3 = M_r = M_N = 0[N \cdot m]$.

As with the previous case from fig.4 one may notice that the $x_m = b_m$ equilibrium points of the linerized system are asymptotically stable resulting that in certain vicinity these points are asymptotically stable for the non-linear system defined by (1).

Next, we will emphasize the influence of the estimated rotor resistance when studying asymptotic stability of the equilibrium point $x_0 = b_0$ of the non-linear system (1) in a certain vicinity of this point.

The equilibrium point is obtained for an entry vector like $u = [u_1 u_{20} u_3]^T$ defined by (2), namely for a speed of $n_0 = 0[rpm]$.

Based on this study we can obtain information related to the range of variation of the estimated rotor resistance for which the non-linear system (1) remains asymptotically stable around the $x_0 = b_0$ equilibrium point.

As the self values of the A_L matrix are presented in fig.5, it results that the non-linear system (1) around the equilibrium point $x_0 = b_0$ is asymptotically stable for an es-

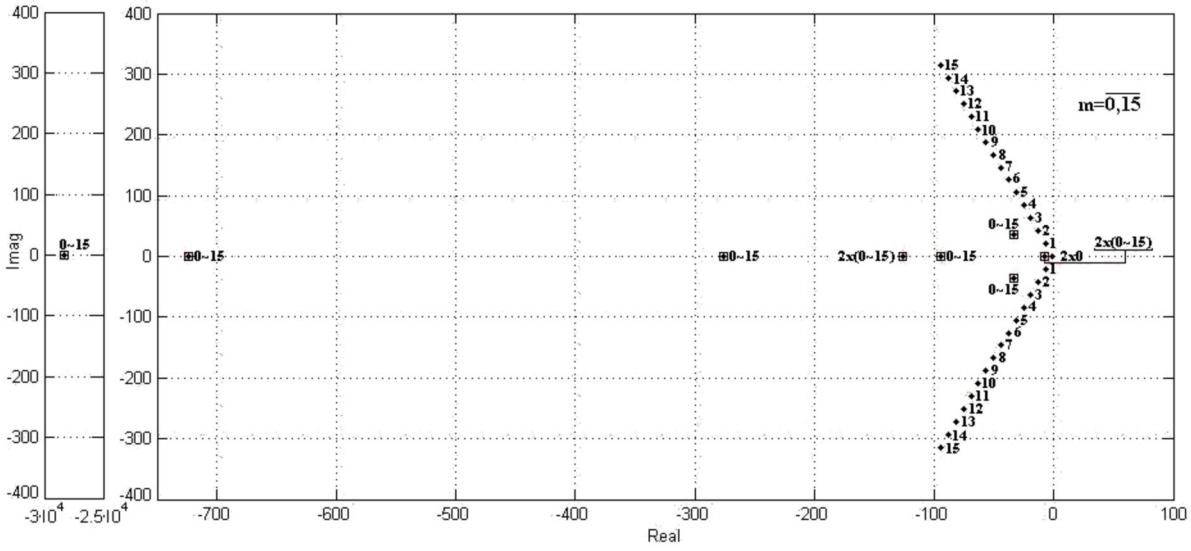


Fig.4. The self values of the matrix A_L; M_r=0

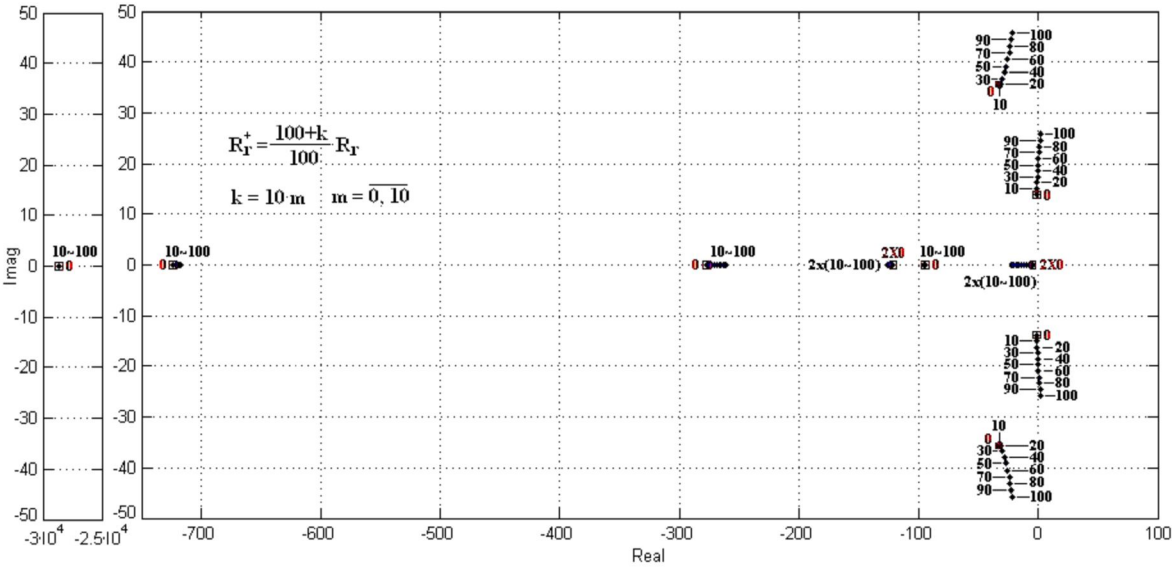


Fig.5. A_L self values by rotor resistance for n0=0[rpm] $Mr=M_N$

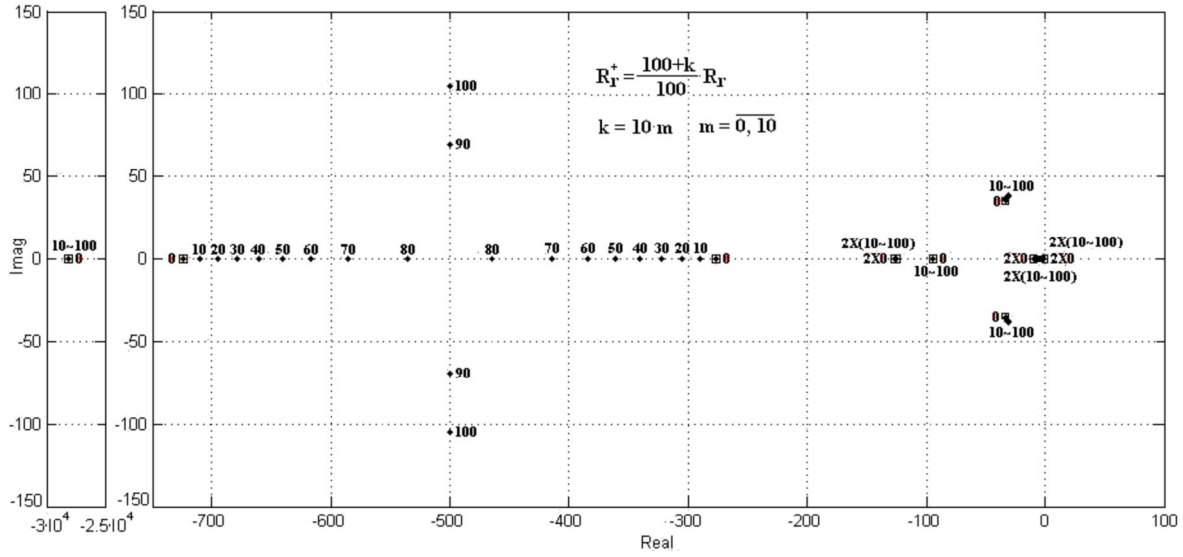


Fig.6. A_L self values by rotor resistance for n_0=0[*rpm*] M_r=*0*

timated rotor resistance in range of $D_1 = \{R_r^*; 0{,}91R_r \le R_r^* \le 1{,}48R_r\}$, becoming asymptotically unstable for an estimated rotor resistance in range of $D_2 = \{R_r^*; 1{,}48R_r \le R_r^* \le 0{,}91R_r\}$.

Next we will present the influence of the estimated rotor resistance to the asymptotic stability of the non-linear system (1) when the entry vector is $u = [u_1 u_{20} u_3]^T$ defined by (2) with the difference that $u_3 = M_r = M_N = 0[N \cdot m]$. The self values of the A_L matrix are presented in fig.6.

As the self values of the A_L matrix are presented in fig.6, it results that the non-linear system (1) around the equilibrium point x_0=b_0 is asymptotically stable for an estimated rotor resistance in range of $D_1 = \{R_r^*; 0{,}1R_r \le R_r^* \le 2{,}61 \cdot 10^6 R_r\}$, becoming asymptotically unstable for an estimated rotor resistance in range of $D_2 = \{R_r^*; 2{,}61R_r \le R_r^* \le 0{,}1R_r\}$.

On the other hand in case we realize the digitization of the (3) linear system we obtain:

$$x(k+1) = F_L x(k) + H_L u(k), \qquad (4)$$

where the F_L, H_L matrixes are obtained from the A_L, B_L matrixes using one of the two digitization types:

- simplified digitization:

$$F_L = I_{12} + A_L T;\ H_L = B_L T; \qquad (5)$$

- complete digitization:

$$F_L = I_{12} + A_L T + A_L^2 \frac{T^2}{2};$$

$$H_L = B_L T + A_L B_L \frac{T^2}{2},$$

where: T is the sampling time.

Proceeding in a similar manner the self values of the F_L matrix in case the entry vector is defined by (2) and the F_L matrix is obtained by using simplified digitization using a $T = 53{,}3[\mu\text{sec}]$ sampling time were considered.

From this consideration we can see, that the x_m=b_m equilibrium points are asymptotically stable for the discrete system (4) in case the F_L matrix is obtained through using the simplified digitization method.

This conclusion also remains valid when the F_L matrix is obtained through the complete digitization method both where the resistant couple is present or absent.

Next we will present the influence of the estimated rotor resistance over the asymptotic stability of the discrete non-linear system when the entry vector is $u = [u_1 u_{20} u_3]^T$ and is defined by (2). This study was conducted for a sampling time of $T = 53{,}3[\mu\text{sec}]$ in case of simplified digitization. After the study we obtained the information that the x_0=b_0 equilibrium point remains asymptotically stable for the (5) linear system for a variation of identi-

fied rotor resistance ranging from $D_1 = \{R_r^*; 0{,}91R_r \le R_r^* \le 1{,}48R_r\}$.

The $x_0=b_0$ equilibrium point becomes asymptotically unstable when the variation of the identified rotor resistance varies from $D_2 = \{R_r^*; 1{,}48R_r \le R_r^* \le 0{,}91R_r\}$.

When the study of the influence of identified rotor resistance variation is done for an entry vector of the $u=[u_1u_2u_3]^T$ type defined by (2) with the difference that $u_3 = M_r = M_N = 0[N\cdot m]$ then the stability domain for which the $x_0=b_0$ equilibrium point is asymptotically stable is $D_1 = \{R_r^*; 0{,}1R_r \le R_r^* \le 306{,}39R_r\}$ and the instability domain is $D_2 = \{R_r^*; 306{,}39R_r \le R_r^* \le 0{,}1R_r\}$.

On the other hand the domains that define the upper and lower limits of variation of identified rotor resistance for which the $x_0=b_0$ equilibrium point remains asymptotically stable for the (5) discrete linear system diminishes with the increase of the sampling time.

4. Conclusions. In this paper is presented under a unitary entity the mathematical model of the speed vector control system that has in its loop a Gopinath observer. This mathematical model allows for external stability study as well as internal stability study of the control system.

After the study we observed that the analyzed regulating system is asymptotically stable for the imposed speed range (0…1500)[*rpm*] both, when a resistant couple is lacking or present as $M_r=M_N$. This conclusion is valid in both the continual and discrete cases where the sampling time is $T = 53{,}3[\mu\text{sec}]$.

From those presented above we can say that the Gopinath observer adds self values to the control system that does not contain it in its loop without modifying the other self values. This conclusion is only valid when the matrixes that are the basis of projecting the Gopinath observer are the same size with those of the induction motor.

In this paper we determined the upper and lower variation limits for the identified rotor resistance for which the $x_0=b_0$ equilibrium point remains asymptotically in both the discrete and continual cases.

After the study we obtained the following information emphasized within Table 1 for the continual case ant Table 2 for the discrete case.

Table 1

Continual case		
	$M_r=M_N$	$M_r=0$
LI	$0{,}91R_r$	$0{,}1\ R_r$
LS	$1{,}48R_r$	$2{,}61\cdot10^6R_r$

Table 2

Discrete case		
	$M_r=M_N$	$M_r=0$
LI	$0{,}91R_r$	$0{,}1\ R_r$
LS	$1{,}48R_r$	$306{,}39R_r$

Where, through *LI* we noted the lower limit and through *LS* we noted the upper limit of variation of identified rotor resistance.

From the tables above one may notice that the upper limit for the discrete case decreases compared to the upper limit of the continual case when the resistant couple is null. The upper limit stays the same in both the discrete and continual cases when $M_r=M_N$.

The lower limit for the discrete case is the same as the lower limit of the continual case in both the lack or presence of the resistant couple.

Also, from the tables above one may notice that the upper limit in case of lacking resistant couple increases compared to the upper limit in case of having resistant couple and the upper limit in case of lacking the resistant couple decreases compared to the lower limit in case of having resistant couple. This conclusion remains valid for both, the continual and discrete cases.