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MONITORING ON-LINE BY THE STABILITY OF THE KALMAN FILTER USED IN THE VECTORIAL CONTROL SYSTEM FOR THE SPEED OF AN INDUCTION MOTOR

In the paper we present the stability study of the Kalman estimator used in a vectorial drive system of an induction motor with short-circuited rotor. We consider that the control system for speed is a vectorial system with direct orientation after the rotor flux (DFOC). Although there are suggestive experimental results, the implementation of such a system containing a Kalman filter generates serious problems because of the tuning of the covariant matrixes of the measurement noise and the process noise. Also the solving of the Riccati equations in real time is difficult. These problems, as well as the simplifier hypothesis from the determination of the mathematical model on the states space of the induction motor and the modification of the rotor resistance by temperature lead to the decrease of the estimator and control system performances. Therefore you must implement an algorithm for research of the estimator stability. This algorithm has to hold count of the sampling step of sundries processors used in the drives command. The stability study is done by the determination of the transition matrix poles.

Представлено устойчивое обучение оценщика Калмана, используемого в векторных системах с асинхронными двигателями и короткозамкнутым ротором. Предполагаем, что система управления скоростью является векторной ориентированной по потоку ротора. Хотя уже существуют экспериментальные результаты, осуществление такой системы с фильтром Калмана вызывает серьезные проблемы из-за необходимости настройки ковариантных матриц измеренных помех и помех, возникающих в процессе работы. Также затруднено решение уравнений Риккати в реальном времени. Эти проблемы также, как упрощенное представление математической модели в пространстве состояний асинхронного двигателя и изменение сопротивления ротора при изменении температуры, приводят к ухудшению качества работы оценщика и системы управления. Поэтому необходимо реализовывать алгоритм исследования устойчивости работы оценщика. Алгоритм должен поддерживать шаг выборки различных процессоров, используемых при управлении двигателями. Устойчивое обучение реализовано путем определения полюсов матрицы перехода состояний.

1. The Kalman filter. The equations which define the recursive algorithm of the Kalman filter where written on the basis set by the following stochastic model of the induction motor:

$$\begin{cases} x(k+1) = F_k x(k) + H_k u(k) + w(k); \\ y(k) = Cx(k) + v(k), \end{cases}$$

where w and v are the process noise vector and the measurement noise vector which are of Gaussian type and have the following properties:

$$\begin{cases} E[w(k)] = E[v(k)] = E[w(i)v^T(j)] = 0; \\ E[w(i)w^T(j)] = Q_k \delta_{ij}; \\ E[v(i)v^T(j)] = R_k \delta_{ij}, \end{cases}$$

where E is a statistic which means:

$$E(x) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} x(k)$$

and δ_{ij} is a Kronecker function:

$$\delta_{ij} = \begin{cases} 1 & \text{for} \quad i = j; \\ 0 & \text{for} \quad i \neq j. \end{cases}$$

The F_k and H_k matrixes are obtained from the A and B matrixes of the continuous model of the induction motor after discrimination. In practice two models are used:

- The model obtained from complete discrimination for which the Kalman filter is precise, but a high performance processor is required for implementation:

$$\begin{cases} F_k = I + A_k T + A_k^2 \dfrac{T^2}{2}; \\ H_k = BT + A_k B \dfrac{T^2}{2}. \end{cases} \quad (1)$$

• The model obtained from simplified discrimination for which the Kalman filter is not so precise, but there is no need for a high performance processor for implementation:

$$\begin{cases} F_k = I + A_k T; \\ H_k = BT. \end{cases} \quad (2)$$

In (1) and (2) T is the sampling period T=(50÷30)μs and A_k and B have forms which depend on the type of the observer:

• **Kalman filter:** The Kalman filter used for the determination of the rotoric fluxes:

$$A_k = \begin{bmatrix} a_{11} & 0 & a_{13} & a_{14}\omega_k \\ 0 & a_{11} & -a_{14}\omega_k & a_{13} \\ a_{31} & 0 & a_{33} & -\omega_k \\ 0 & a_{31} & \omega_k & a_{33} \end{bmatrix};$$

$$B = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{11} & 0 & 0 \end{bmatrix}^T; \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad (3)$$

where:

$$a_{11} = -\left(\frac{1}{T_s\sigma} + \frac{1-\sigma}{T_r\sigma}\right); \; a_{13} = \frac{L_m}{L_s L_r T_r \sigma};$$

$$a_{14} = \frac{L_m}{L_s L_r \sigma}; \quad a_{31} = \frac{L_m}{T_r}; \quad a_{33} = -\frac{1}{T_r};$$

$$b_{11} = \frac{1}{L_s\sigma}; \quad T_s = \frac{L_s}{R_s}; \quad T_r = \frac{L_r}{R_r}; \quad \sigma = 1 - \frac{L_m^2}{L_s L_r}$$

and L_s, L_r, L_m are the stator, rotor and mutual inductance, R_s, R_r are the stator and rotor resistance and σ is the mutual coefficient of leakage.

The matrixes (3) were determined for:

• the elements of the input vector are the stator voltages reported to the orientated axis system *dq*:

$$u(k) = \begin{bmatrix} u_{ds}(k) & u_{dq}(k) \end{bmatrix}^T;$$

• the elements of the state vector are the stator currents and the rotor fluxes reported to the orientated axis system *dq*:

$$x(k) = [i_{ds}(k) i_{dq}(k) \psi_{dr}(k) \psi_{qr}(k)]^T;$$

• the elements of the output vector are the stator currents reported to the orientated axis system dq:

$$y(k) = \begin{bmatrix} i_{ds}(k) & i_{dq}(k) \end{bmatrix}^T.$$

• **The extended Kalman filter:**

The extended Kalman filter matrixes used for the determination of the rotor fluxes and of the rotor speed are:

$$A_k = \begin{bmatrix} a_{11} & 0 & a_{13} & a_{14}\omega_k & 0 \\ 0 & a_{11} & -a_{14}\omega_k & a_{13} & 0 \\ a_{31} & 0 & a_{33} & -\omega_k & 0 \\ 0 & a_{31} & \omega_k & a_{33} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{11} & 0 & 0 \end{bmatrix}^T; \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \quad (4)$$

The matrixes (4) were determined for:

• the elements of the input vector are the stator voltages reported to the orientated axis system *dq*:

$$u(k) = [u_{ds}(k) u_{dq}(k)]^T;$$

• the elements of the state vector are the stator currents and the rotor fluxes reported to the orientated axis system *dq* and the rotor speed:

$$x(k) = [i_{ds}(k) i_{dq}(k) \psi_{dr}(k) \psi_{qr}(k) \omega(k)]^T;$$

• the elements of the output vector are the stator currents reported to the orientated axis system *dq*:

$$y(k) = [i_{ds}(k) i_{dq}(k)]^T.$$

In those conditions, the equations which define the Kalman filter are:

a) The Kalman filter:

$$\tilde{A}(k/k-1) = F_{k-1} P(k-1/k-1) F_{k-1}^T + Q_{k-1};$$

$$K(k) = \tilde{A}(k/k-1) C^T [\tilde{N}\tilde{A}(k/k-1) C^T + R_k]^{-1};$$

$$\hat{x}(k/k-1) = F_{k-1}\hat{x}(k-1/k-1) + H_{k-1}u(k-1); \quad (5)$$

$$\hat{x}(k/k) = \hat{x}(k/k-1) + K(k)[y(k) - C\hat{x}(k/k-1)];$$

$$P(k/k) = [I_4 - K(k)C]\tilde{A}(k/k-1),$$

where:

- $(k/k-1)$ is an estimate at the moment kT on the basis of the anterior data which is simultaneous at the moment $(k-1)T$;
- $K(k)$ is the Kalman matrix;
- $\hat{x}(k/k-1)$ is the extrapolated state (provided);
- $\hat{x}(k/k)$ is the estimated state at the moment kT;
- $y(k)-C\cdot\hat{x}(k/k-1)$ is the estimate of the extrapolated state;
- $\Gamma(k/k-1)$ is the covariance a priori matrix of the extrapolated state $\hat{x}(k/k-1)$;
- $P(k/k)$ is the covariance a posteriori matrix of the estimated state $\hat{x}(k/k)$.

The estimate error is:

$$\tilde{x}(k/k)=x(k)-\hat{x}(k/k). \qquad (6)$$

The initial conditions of the algorithm (5) are:

$$P(0/0)=P_0 \quad \text{and} \quad \hat{x}(0/0)=\hat{x}_0.$$

The covariance matrixes Q and R are constant or are updated at every step. In the paper we consider that the noises do not sensibly modify the process and they could be considered constants. Accordingly, the matrixes Q and R are:

$$R=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix};\quad Q=\begin{bmatrix}\alpha_1 & 0 & \alpha_3 & 0\\ 0 & \alpha_1 & 0 & \alpha_3\\ \alpha_3 & 0 & \alpha_2 & 0\\ 0 & \alpha_3 & 0 & \alpha_2\end{bmatrix},$$

where:

- $\alpha_1=\dfrac{\sigma_{\psi r}}{\sigma^2};\quad \alpha_2=\dfrac{\sigma_{is}}{\sigma^2};\quad \alpha_3=\dfrac{\rho\sigma_{\psi r}\sigma_{is}}{\sigma^2};$
- σ is the leakage introduced by the input variables (u_{ds} and u_{dq});
- σ_{i_s} and σ_{ψ_r} are the leakages introduced by the output variables (i_s and ψ_r);
- ρ is the correction coefficient.

Because the PWM inverter introduced the superior harmonics in the voltage and the stator current components and, implicit, in the rotor flood tides component, we consider for simulation a source of white noise (Gaussian white noise in discreet time) $K(m, n)=\sigma_2\delta_{mn}$; where δ_{mn} is a Kronecker function.

On the other side, the vector $\hat{x}_0$ we consider that it is null and the matrix P_0 we determined as solution of discreet Riccati equation of filtration.

So as determine accurate values of the parameters α_1; α_2 and α_3, we define an application – dependent operating mode and we minimize the mean square error of the rotor flux modulus for the case rotor resistance uncertainty: $\Delta Rr/Rr=\pm 60[\%]$:

$$J(\alpha_1\alpha_2\alpha_3)+\sqrt{\frac{1}{N}\sum_{k=1}^{N}[\psi_r(k-\hat{\psi}_r(k)]^2},$$

where: $\hat{\psi}_r(k)=\sqrt{\hat{\psi}_{dr}^2(k)+\hat{\psi}_{qr}^2(k)}$.

b) The extended Kalman filter:

In order for the structural properties of the Kalman filter to be satisfied we will introduce a virtual speed ω_m defined $\omega_m(k+1)=\omega_m(k)+v_6(k)$ so that we'll have:

$$F_{k-1}=\begin{bmatrix}F_{k-1} & G_{k-1}\\ 0_{2x4} & I_2\end{bmatrix}$$

where:

$$G_{k-1}=\begin{bmatrix}f_1 & f_2 & f_3 & f_4\\ -f_2 & f_1 & -f_4 & f_3\end{bmatrix}^T;$$

$$\frac{\partial}{\partial\omega}\left(F_{k-1}x(k-1)+H_{k-1}u(k-1)\right)=\begin{bmatrix}f_1 & f_2 & f_3 & f_4\end{bmatrix}^T$$

and F_{k-1}, H_{k-1} from inside the resented matrix are obtained from (3) by discrimination using (1) and (2).

The covariance matrixes Q and R are:

$$R=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix};\quad Q=\begin{bmatrix}Q & 0_{2\times4}\\ 0_{2\times4} & \alpha_4 I_2\end{bmatrix},$$

where: $\alpha_4=\sigma_\omega/\sigma_4$; σ_ω is the leakage speed.

- C and H_{k-1} matrixes that interfere in the algorithm of the extended Kalman filter are obtained by the discrimination using (1) and (2) from:

$$B=\begin{bmatrix}b_{11} & 0 & 0 & 0 & 0 & 0\\ 0 & b_{11} & 0 & 0 & 0 & 0\end{bmatrix};$$

$$\tilde{N}=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\end{bmatrix}.$$

Continuing, the extended Kalman filter is similar with (5) with the observations presented.

2. The determination of the K_c matrix. For the stability study of the Kalman filter and of the extended Kalman filter we need the K_c matrix that makes the connection between the size entireness of the $u(k)$ motor and the vector of the estimated sizes $x(k)$.

So:

$$u(k)=-K_c(k)\hat{x}(k), \qquad (7)$$

where:

$$K_c(k)=\begin{bmatrix}0 & 0 & -\dfrac{u_{sd\lambda}(k)}{|\psi_r(k)|} & \dfrac{u_{sq\lambda}(k)}{|\psi_r(k)|}\\ 0 & 0 & -\dfrac{u_{sq\lambda}(k)}{|\psi_r(k)|} & -\dfrac{u_{sd\lambda}(k)}{|\psi_r(k)|}\end{bmatrix}$$

for Kalman filter; and

$$K_c(k)=\begin{bmatrix}0 & 0 & -\dfrac{u_{sd\lambda}(k)}{|\psi_r(k)|} & \dfrac{u_{sq\lambda}(k)}{|\psi_r(k)|} & 0 & 0\\ 0 & 0 & -\dfrac{u_{sq\lambda}(k)}{|\psi_r(k)|} & -\dfrac{u_{sd\lambda}(k)}{|\psi_r(k)|} & 0 & 0\end{bmatrix}$$

for the extended Kalman filter.

Voltages $u_{sd\lambda}(k)$ and $u_{sq\lambda}(k)$ and also flux rotor absolute $|\psi_r(k)|$ are obtained based to the appendix 1 and 2.

3. Stability study by neglecting Kalman filter dynamics. The start relation is:

$$x(k+1)=F_k x(k)+H_k u(k)+w(k).$$

Replacing the expression from (6) and (7) in this relation, it is obtained:

$$x(k+1)=[F_k-H_kK_c(k)]x(k)+H_kK_c(k)\tilde{x}(k/k)+w(k).$$

For a first approximation, it is assumed that the error is neglected $\tilde{x}(k/k)\cong 0$ and then results:

$$x(k+1)=[F_k-H_kK_c(k)]x(k)+w(k).$$

Using z coupled impedance and assuming the process noise null, $w(k)=0$ results:

$$[zI-(F_k-H_kK_c(k))]X(z)=0.$$

So, for the closed-loop to be a stable system, specific values of $(F_k-H_kK_c(k))$ have to be in basic circle around z flat origin.

Generally, this procedure supposes that there is no Kalman filter associated dynamics, which is not true.

4. Stability study when the loop is contained by Kalman filter. At large the motor does not have the same transition matrix like those used for design work of Kalman filter and K_c control matrix. It is possible that the built-in Kalman filter model and motor to be different. Because of this, we assume that the motors matrixes are $F(nxn)$; $H(nxp)$; $C(qxn)$ and the equivalent model matrix are: $F^*(n_r xn_r)$; $H^*(n_r xp)$; $C^*(qxn_r)$. The control matrix is K_c (pxn_r) and the Kalman matrix is $K(n_r xq)$.

The system equations are:

- the motor:

$$x(k+1)=F_k x(k)+H_k u(k)+w(k)$$

- the control law:

$$u(k-1)=-K_c(k-1)\hat{x}(k-1/k-1)$$

- estimation:

$$\hat{x}(k/k)=\Delta(k-1)\hat{x}_r(k-1/k-1)+ +K(k)Cx(k)+K(k)v(k)$$

where:

$$\Delta(k-1)=\left[I-K(k)C^*\right]\left[F^*_{k-1}-H^*_{k-1}K_c(k-1)\right]$$

The (8) relation can be written differently:

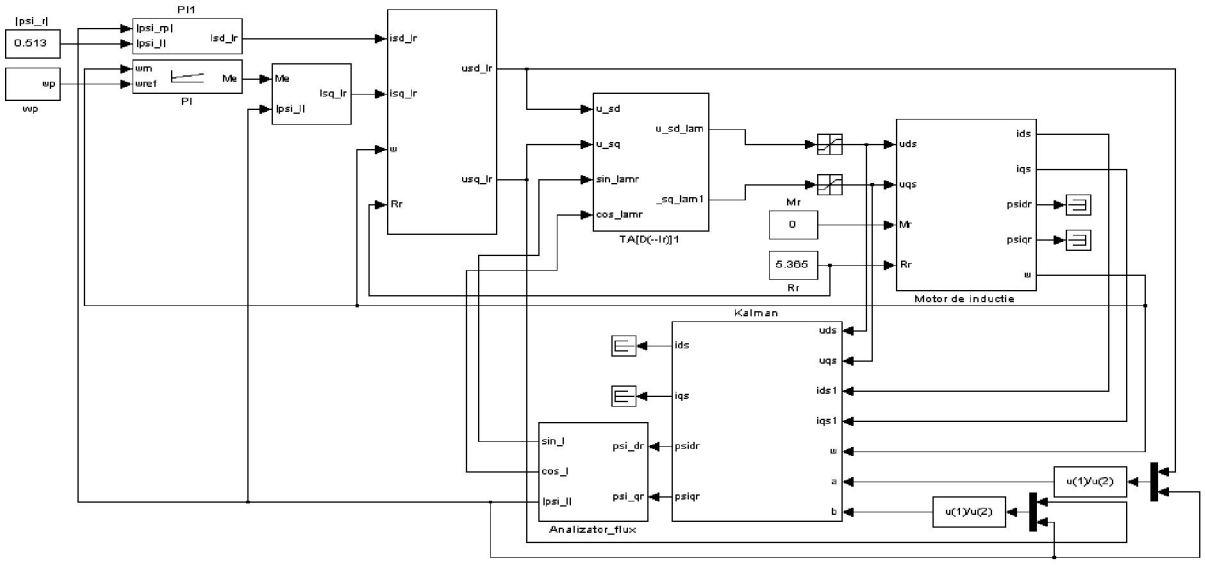
$$\hat{x}_r(k/k)=K(k)CF_{k-1}x(k-1)+ +[\Delta(k-1)-K(k)CH_{k-1}K_c(k-1)]\times \times\hat{x}(k-1/\mathrm{k}-1)+K(k)Cw(k-1)+K(k)v(k). \qquad (9)$$

On the other hand:

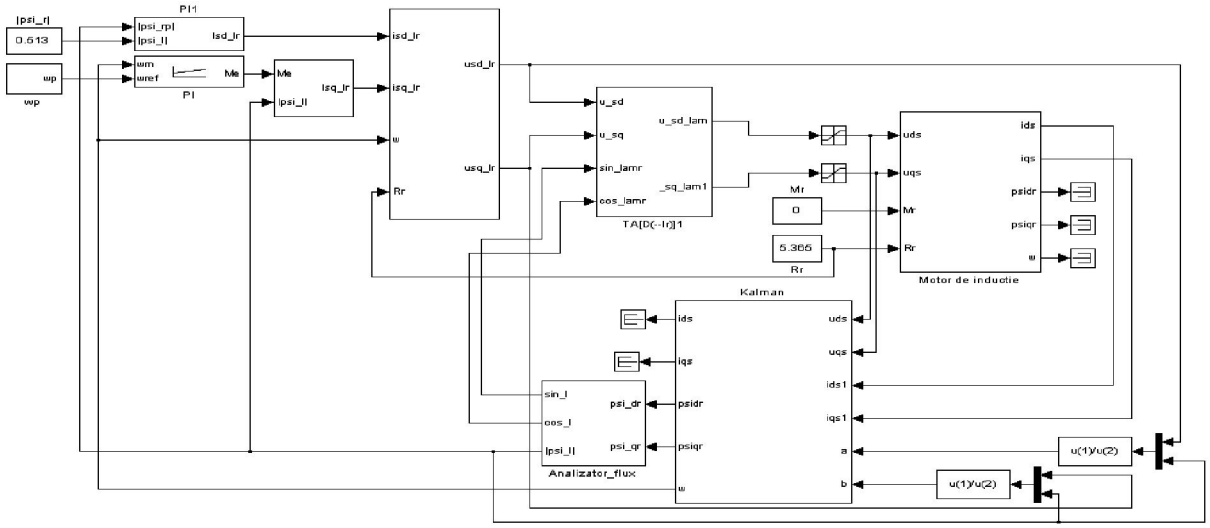
$$x(k)=F_{k-1}x(k-1)-H_{k-1}K_c(k-1)\times \times\hat{x}_r(k-1/k-1)+w(k-1). \qquad (10)$$

The (9) and (10) relations can be matrix write:

$$\begin{bmatrix}x(k)\\ \hat{x}(k/k)\end{bmatrix}=\begin{bmatrix}F_{k-1} & -H_{k-1}K_c(k-1)\\ K(k)CF_{k-1} & \Delta(k-1)-K(k)CH_{k-1}K_c(k-1)\end{bmatrix}\times \times\begin{bmatrix}x(k-1)\\ \hat{x}(k-1/k-1)\end{bmatrix}+\begin{bmatrix}1\\ K(k)C\end{bmatrix}w(k-1)+\begin{bmatrix}0\\ K(k)\end{bmatrix}v(k).$$



Appendix 1



Appendix 2

Because w and v are Gauss uniform rows with zero average autonomous of $[x(0) \quad \hat{x}_r(0/0)]^T$, then $[x(k) \quad \hat{x}_r(k/k)]^T$ is a zero average Gauss-Markov row and the system dynamics is ruled by the specific values of the square transition matrix $F_A(n+n_r xn+n_r)$:

$$F_A = \begin{bmatrix} F_{k-1} & -H_{k-1} \cdot K_c(k-1) \\ K(k)CF_{k-1} & \Delta(k-1) - K(k)CH_{k-1}K_c(k-1) \end{bmatrix}.$$

5. Special case when the motor and the model are similar. If the motor and model have the same order, then $\hat{x}_r$ became $\hat{x}$ and $x = \hat{x} + \tilde{x}$. The same:

$$\Delta(k-1) - K(k)CH_{k-1}K_c(k-1) =$$

$$= [I - K(k)C^*]F^*_{k-1} - H^*_{k-1}K_c(k-1) +$$

$$+ K(k)[C^*H^*_{k-1} - CH_{k-1}]K_c(k-1).$$

The $[x \quad \hat{x}]^T$ vector can be transformed in $[x \quad \tilde{x}]^T$ vector by transforming:

$$\begin{bmatrix} I & 0 \\ I & -I \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} = \begin{bmatrix} x \\ \tilde{x} \end{bmatrix}.$$

By application of this transformation of the transition matrix and if the transforming matrix is its personal reverse matrix we have:

$$\begin{bmatrix} I & 0 \\ I & -I \end{bmatrix} \begin{bmatrix} F_{k-1} & -H_{k-1}K_{\tilde{n}}(k-1) \\ K(k)CF_{k-1} & \Delta(k-1)-K(k)\tilde{N}H_{k-1}K_c(k-1) \end{bmatrix} \times$$
$$\times \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix} = \begin{bmatrix} F_{k-1} _ H_{k-1} & H_{k-1}K_c(k-1) \\ Y & Z \end{bmatrix}$$

where:

$$Y = (I - K(k)C)F_{k-1} - (I - K(k)C^*)F^*_{k-1} +$$
$$+ (H^*_{k-1} - H_{k-1})K_c(k-1) - K(k) \times$$
$$\times (C^*H^*_{k-1} - CH_{k-1})K_c(k-1);$$
$$Z = (I - K(k)C^*)F^*_{k-1} - (H^*_{k-1} - H_{k-1}) \times$$
$$\times K_c(k-1) - K(k)(C^*H^*_{k-1} - CH_{k-1})K_c(k-1).$$

When Kalman filter model and installation are the same, that is $F = F^*$; $H = H^*$; $C = C^*$ then $Y = 0$ and $Z = (I - K(k)C)F_{k-1}$.

6. Simulation results. As an example for what was mentioned, an induction motor with the circuited rotor that has the fallowing parameters:

$$P_n = 500[W]; U_n = 127[V]; I_n = 2{,}9[A];$$
$$n_n = 1400[rot/\min]; z_p = 2; M_n = 3{,}41[N \cdot m]$$
$$R_s = 4{,}495[\Omega]; R_r = 5{,}365[\Omega]; L_s = 165[mH];$$
$$L_r = 162[mH]; L_m = 149[mH]$$
$$J = 0{,}95 \cdot 10^{-3}[Kg \cdot m^2]; f_n = 50[Hz]$$

has been considered. In order to obtain the effects of the use of the Kalman filter in feedback with an induction motor of the type mentioned, the mathematical model of the motor and also the one of the Kalman filter have been implemented in Matlab-Simulink, using the S_Function blocks. The sampling time used is $300[\mu s]$. In this simulation it has been considered that the $K(k)$ matrix is variable and it actualizes at every step.

7. Conclusions. Luenberger has studied the problem of the estimators included in the continual linear control systems. He showed that the estimator adds poles to the feed-back system without affecting the others poles. Due to the simulation of the three cases presented we showed that this is true only when the estimator is made to give a model to the motor (the last case presented), even if this is not an optimal project (in stochastic sense). After the analysis, it has been noticed that at speeds close to the nominal speed and also at very small speeds, eigenvalues of the estimator are very close of the ones of the motor, and we can conclude that the control system works fine. Bigger differences appear only at high speed. After the analysis it also has been noticed that part of the eigenvalues of the estimator fall behind the eigenvalues of the motor that practically means a decrease of the control system performances. The speed limit when this phenomenon appears depends of the sampling step, diminishing in the same time with the increase of the latter. While using the Kalman filters in vectorial control systems for speed, the fact that they become unstable when using a big sampling step, which means a decrease in the control system performances must be take in account.