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ОЦЕНКА КРИТЕРИЕВ РАЗРУШЕНИЯ ДЛЯ ЧИСЛЕННЫХ МОДЕЛЕЙ ГОРНОГО МАССИВА

Работа представляет результаты численного моделирования с применением пластических конструктивных моделей с различными критериями разрушения. На языке FISH были разработаны критерии разрушения Балмера и Баландина и применены для моделирования двух основных проблем: сжатие горного образца и сооружение конструкций в горном массиве.

Ключевые слова: численное моделирование, критерии разрушения, геомеханика, подземные разработки

IMPLEMENTATION OF VARIOUS FAILURE CRITERIA FOR NUMERICAL MODELS OF ROCK MASS

The paper presents results of numerical modeling by application of plastic constitutive models with various failure criteria. The Balmer's and Balandin's failure criteria have been elaborated in FISH language and they have been applied to simulate two basic problems which are: compression of rock sample and construction of excavation in rock mass.

Key words: numerical modeling, failure criteria, rock mechanic, underground excavations

1. Introduction.

FLAC program is based on finite difference method which is one of the most common tools used for numerical modeling for issues in the scope of rock mechanic and geotechnics.

This program allows for construction of numerical models throughout application of different constitutive models with taking into consideration adequate properties of rock mass. Material properties to be adopted depend on selected constitutive models. The essential constitutive models, which are initially applied to the program (Itasca, 2002) could be divided into two basic groups which are: elastic and plastic with failure criteria of Drucker-Prager, Hook-Brown and Coulomb-Mohr. The Coulomb-Mohr's criterion also applies to other basic plastic models available in the program, which are:

- strain-hardening/softening model;
- ubiquitous-joint model;
- bilinear strain-hardening/softening ubiquitous-joint model;
- double-yield model.

One of the main feature of this program is its flexible formula which allows a user for creation of optional constitutive models with assistance of the internal programming language (FISH) as well as with usage of DLL files (dynamic-link library), which make this program very valuable not only for solving engineering problems, but also for more advanced researches.

Many of constitutive models which are created by users have been presented during five recent conferences on numerical modeling with assistance of FLAC program. These conferences are organized by the Itasca Consulting Group which is the program manufacturer.

Despite of rich range of constitutive models primarily implemented in FLAC, conducting calculations with applying another strength criterion could provide more correct results. Although, it is necessary to pay attention to wide variety and a number of up to date elaborated strength criteria, for example publication by M.Kwaśniewski (2002) has gathered about 50 different failure criteria for granite.

Back analysis is one of the methods of acquiring information on materials' features (for example for rock mass) on the basis of in situ measurements. Sakurai (1997) emphasises that mechanical model shouldn't be determined before commencing calculation for issues associated with mechanism of rock mass, but it also should be established by applying back analysis. Possibility of implementation of various failure criteria for FLAC program could be helpful for conducting this type of calculation. However, knowledge of mechanical model and its applicable parameters is required for calculation of stress and deformation in the vicinity of mining excavation and as well as conditions of cooperation of support and lining with rock mass (Chudek, 2010).

The results of basic calculation have been presented in this article. This calculation has been carried out with application of the algorithms written in FISH language which have been established which implementation of Balmer's (1952) and Balandin's (1937) failure criteria.

2. Implemented failure criteria.

The algorithms established with assistance of FISH (FLAC internal programming language) allow for modeling plastic material by applying Balmer's and Balandin's failure criteria. In both cases, failure criterion has been defined by uniaxial tensile and compressive strength.

However, the first of them is an empirical criterion which assumes that value of intermediate principal stress has no influence on rock strength, the second one takes into account all three principal stresses. Algorithms which include plastic models with the above failure criteria, have been elaborated throughout modification of Coulomb-Mohr's constitutive model in FISH language.

Firstly, algorithm for Balmer's failure criterion has been elaborated, in which yield surface is defined in a term of major and minor principal stresses by the following formula:

$$\sigma_1 = \sigma_c(1+\frac{\sigma_3}{\sigma_t})^b \text{ and } \sigma_3 = -\sigma_t \qquad (1)$$

where: σ_c – uniaxial compressive strength, σ_t – uniaxial tensile strength, b – factor dependable on rock mass quality.

In order to make calculations, Balmer's model should be applied to elements of finite differences and suitable material features need to be established, such as:

- *ba_k* – elastic bulk modulus,
- *ba_g* – elastic shear modulus,
- *Rc* – uniaxial compressive strength,
- *Rr* – uniaxial tensile strength,
- *ba_b* – factor dependable on rock mass quality,
- *ba_dil* – dilatation angle.

The second failure criterion is a detailed case of Burzyński – Jagna's strength criterion. It has been proposed by Balandin (1937) and it has the following formula:

$$\sigma_1^2+\sigma_2^2+\sigma_3^2-(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)-$$
$$-(\sigma_c-\sigma_t)(\sigma_1+\sigma_2+\sigma_3)=\sigma_c\sigma_t \qquad (2)$$

This criterion in space of principal stresses creates yield surface in the form of paraboloid of revolution. Implementation of this failure criterion for plastic model has been conducted in FLAC program by applying the above function expressed in the convention of octahedral stresses

$$\tau_{oct} = 1/3\sqrt{6(\sigma_c-\sigma_t)\sigma_{oct}+2\sigma_c\sigma_t} \qquad (3)$$

where:

$$\tau_{oct} = 1/3\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}\,; \qquad (4)$$

$$\sigma_{oct} = 1/3(\sigma_1+\sigma_2+\sigma_3). \qquad (5)$$

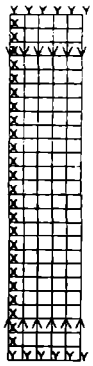


Fig.1. Model of sample at uniaxial compressive test with boundary conditions

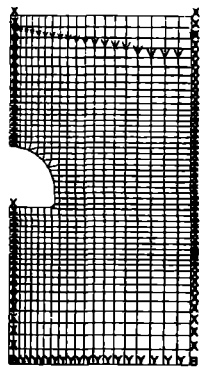


Fig.2. Model of rock mass with excavation and boundary conditions

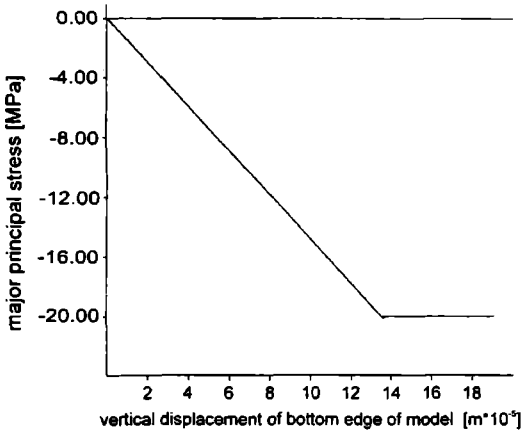


Fig.3. major principal stress registered during simulation of uniaxial compression test for plastic material with Balmer's failure criterion

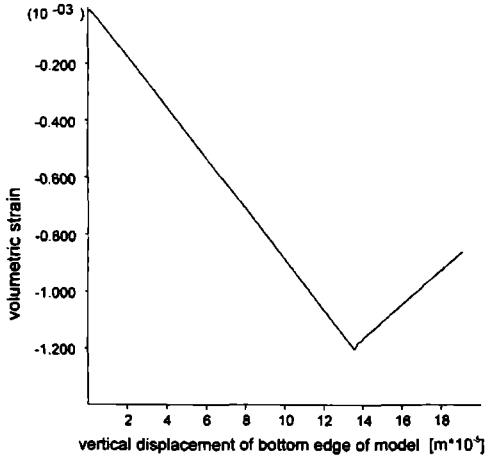


Fig 4. Diagram of volume strain registered during simulation of uniaxial compression test for plastic material with Balmer's failure criterion

Application of this algorithm allows for numerical calculation for Baladin's plastic model after writing a command «*model burz*» and after adapting suitable material features: *burz_k*, *burz_g*, *Rc*, *Rr*, *burz_dil*.

3. Application of compiled algorithms.

Verifications of elaborated algorithms have been conducted by applying two numerical models:

- model of cylindrical rock sample exposed to uniaxial compressing;
- model of rock mass with excavation in shape of ŁP-8 support (the most common type of yield steel arch support which is utilized in Polish coal mining).

3.1. Plastic model with Balmer's failure criterion.

Cylindrical shape model exposed to uniaxial compression.

Axial symmetrical model in cylindrical shape with dimension of 54 mm – diameter and 135mm – height has been applied for sample and simulation of uniaxial compressing (fig.1).

A model shape with accepted boundary conditions and with division into elements of finite differences has been presented in fig.1. The following material features have been accepted for this model: elastic bulk modulus (ba_k) 5,56 GPa, elastic shear modulus (ba_g) 4.17 GPa, uniaxial compressive strength (Rc)-2 MPa, uniaxial tensile strength (Rr) – 2 MPa, dilatation angle 10°, b factor (ba_b) – 0,6.

Model load was established by application of increasing movements of the lower and the upper edge which simulate movements of loading platens. Suitable movements speed of loaded edges has been reached by applying al-

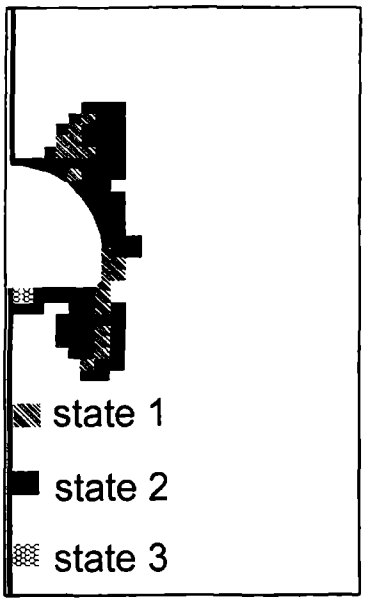


Fig.5. Distribution of plasticity zones in the vicinity of excavation for model with Balmer's failure criterion

gorithm which is named *servo.fis* which has been elaborated by the program's manufacturer (Itasca, 2002). Vertical displacement of the lower edge of the model, principal stresses and volume strain have been registered during simulation. The results of carried out simulations have been presented in a form of a diagram presenting growth of major principal stress (fig.3) and volume strain changes (fig.4) with relation to displacement of the lower edge of the model.

According to the above diagram, constitutive model possesses characteristic of plastic model in which the material after reaching failure, experiences plastic deformation with a stable level of stress. A plastic flow goes through with consideration of dilatation process.

Rock mass model with excavation.

Application of the elaborated algorithm for rock mass modeling with excavation has been presented by simulation of underground roadway in a shape of ŁP support in a flat shield with dimension of 20 m for 10 m. The model shape with division into finite elements differences and boundary conditions has been presented in fig.2.

Boundary conditions have been applied to this model which fixation of displacements of nodes located on the sideway and the lower edge in a perpendicular direction. The upper edge has been loaded with pressure of 25 MPa.

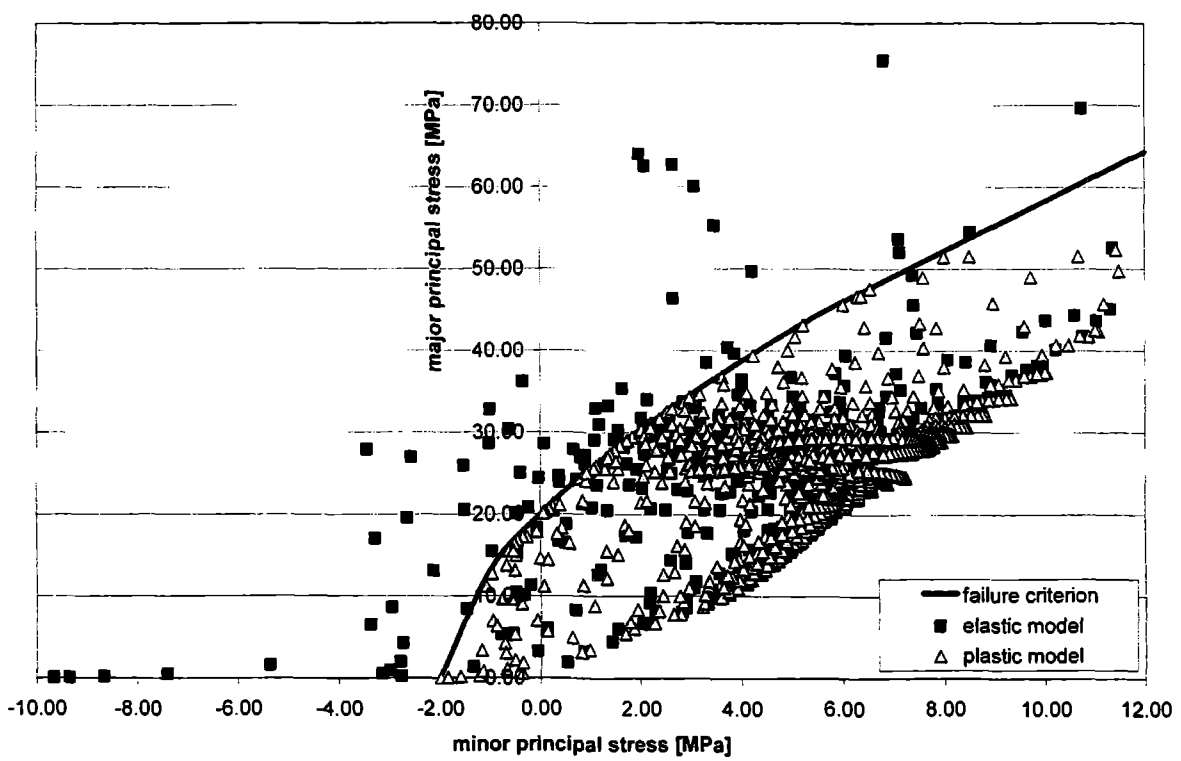


Fig.6. Stress state in modeled rock mass for elastic and plastic model with Balmer's failure criterion

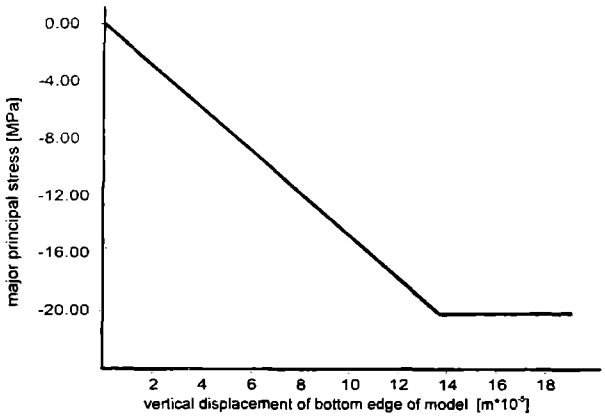


Fig.7. Major principal stress registered during simulation of uniaxial compression test for plastic material with Baladin's failure criterion

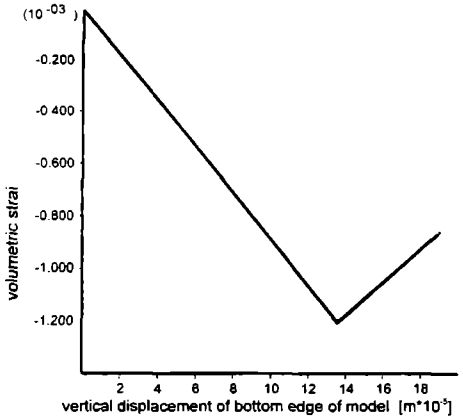


Fig.8. Diagram of volume strain registered during simulation of uniaxial compression test for plastic material with Baladin's failure criterion

Accepted primary conditions reflected initial state of stress, which is: $p_z = 25$ MPa, $p_x = p_y = 6.25$ MPa (p_z – vertical stress, $p_x = p_y$ – horizontal stress). The same material features have been accepted for the whole model as described above for simulation of uniaxial compressing. The calculation results for the model prepared in this manner have been presented in fig.5-6.

Fig.5 presents plasticity zones of rock mass in vicinity of excavation. The elaborated algorithm attributes the symbol of state to each element which constructs the model, which can be shown with assistance of a command "*plot uplast*" The value of numbers have been imputed to this symbol of states:

- 0 – failure criterion has not been reached;
- 1 – failure criterion has been reached in an actual calculation step;
- 2 – failure criterion has not been reached in an actual calculation step but has been reached in the past;
- 3 – tensile strength has been reached.

Fig.6 has been presented in order to demonstrate the influence of accepted failure criterion affecting stress decomposition in the analyzed model. This diagram which presents stress state in space of principal stresses σ_1, σ_3 for elastic model (square indicators) and plastic model (triangular indicators) as well as with the line indicating stress state which satisfies Balmer's strength criterion.

3.2. Plastic model with Balandin's failure criterion.

Cylindrical shape model exposed to uniaxial compression.

Behavior of numerical models with implementation of Balandin's failure criterion has been tested for the same conditions which have been described above in the case of verification of Balmer's model. The same values of material features have been accepted.

Fig.7 presents changes of the major principal stress in simulated process of compressing. This diagram has been agreed to characteristic of plastic material and it shows the strain growth in simulated sample until it reaches the

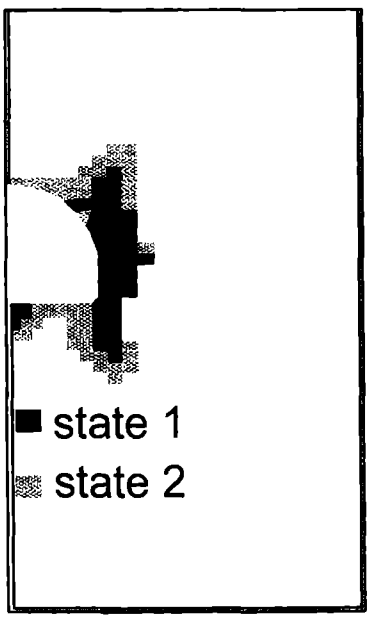


Fig.9. Distribution of plasticity zones in the vicinity of excavation for model with Balandin's failure criterion

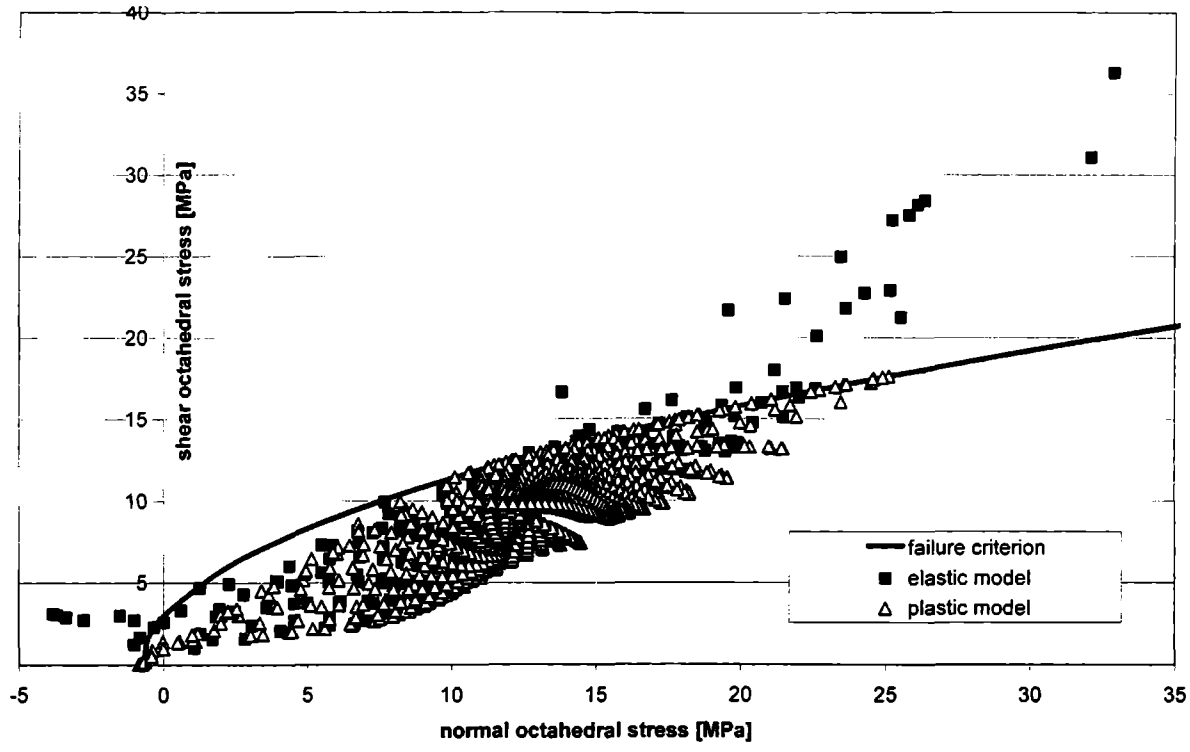


Fig.10. Stress state in convention of octahedral stress for elastic and plastic model with Baladin's strength criterion

uniaxial compressive strength (20 MPa) whereby with the increase of deformations, stress remains on a stable level. Fig.8 presents volume strain which appears in simulated sample. This model behaves in the same way as the one where Balmer's criterion has been applied, which means after reaching failure it meets relative growth of volume strain.

Rock mass model with excavation.

Application of prepared alogorithms has allowed to define plasticity zones of rock mass model with excavation. There are three distinctive states in this model:

- 0 – failure criterion has not been reached;
- 1 failure criterion has been reached in an actual calculation step;
- 2 –failure criterion has not been reached in an actual calculation step, but has been reached in the past.

Fig.9 demonstrates layout of plasticity zones in the model where Balandin's failure criterion has been applied. Comparing plasticity zones gained for both failure criteria, it is necessary to pay attention to their similar extents and differences in their shape which are the results of application of differentiated criteria.

Distribution of stresses in convention of octahedral stress has been also presented in fig.10 with:

- modeling results where plastic model was used with Balandin's criterion (square indicators);
- results of calculation carried out for elastic model (triangular indicators);
- the line indicating stress state which satisfies Baladin's strength criterion.

4. Conclusions.

Two failure criteria have been presented in the first part of this article (Balmer's and Baladin's) for which the algorithms have been elaborated allowing their implementation for plastic model in FLAC program. Subsequently, the results of sample calculation with application of elaborated algorithms have been placed, such as numerical modeling of rock sample exposed to uniaxial compressing and rock mass modeling with excavation.

The modeling results which have proved plastic character of construction material have been presented in the following pictures:

- a diagram of principal major stress changes during attempt of uniaxial compressing – stress were increasing linear for both constitutive

models in the first stage of loading according to accepted Young's model, then the model has experienced further deformation without stress growth after reaching compressive strength;

• a diagram of volume strain changes – in both constitutive models after reaching compressive strength by stress, relative growth of volume could be observed;

• plasticity zones with taking into consideration actual state of elements.

Possibility of applying various failure criteria in numerical model can be defined as a forward step in the direction of improved projection of processes taking place in rock mass with taking into account impact of mining activities. It is important to keep in mind, that even possibility of application of any strength criterion doesn't solve a problem of its selection and defining its parameters.

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