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Investigation of the accuracy of constructing digital elevation models of technogenic massifs based on satellite coordinate determinations

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Abstract

At all stages of the life cycle of buildings and structures, geodetic support is provided by electronic measuring instruments – a laser scanning system, unmanned aerial vehicles, and satellite equipment. In this context, a set of geospatial data is obtained that can be presented as a digital model. The relevance of this work is practical recommendations for constructing a local quasigeoid model and a digital elevation model (DEM) of a certain accuracy. A local quasigeoid model and a DEM were selected as the study objects. It is noted that a DEM is often produced for vast areas, and, therefore, it is necessary to build a local quasigeoid model for such models. The task of assessing the accuracy of constructing such models is considered; its solution will allow obtaining a better approximation to real data on preassigned sets of field materials. A general algorithm for creating both DEM and local quasigeoid models in the Golden Software Surfer is presented. The constructions were accomplished using spatial interpolation methods. When building a local quasigeoid model for an area project, the following methods were used: triangulation with linear interpolation (the least value of the root mean square error (RMSE) of interpolation was 0.003 m) and kriging (0.003 m). The least RMSE value for determining the heights by control points for an area project was obtained using the natural neighbour (0.004 m) and kriging (0.004 m) methods. To construct a local quasigeoid model for a linear project, the following methods were applied: kriging (0.006 m) and triangulation with linear interpolation (0.006 m). Construction of the digital elevation model resulted in the least aggregate value of the estimated parameters: on a flat plot of the earth's surface – the natural neighbour method, for a mountainous plot with anthropogenic topography – the quadric kriging method, for a mountainous plot – quadric kriging.

Keywords

digital elevation model; point cloud; spatial interpolation methods; quasigeoid; normal height; geodetic height; root mean square error; kriging; triangulation; natural neighbour

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Introduction

Geodetic support, which includes engineering and geodetic survey, siting work, and pre-construction survey, is an important aspect of ensuring the design, construction and operation of buildings and structures, including projects in the mineral resources sector. During operation, monitoring of the facility deformations can be carried out [1]. At all life cycle stages of mineral resource sector projects it is possible to use the advanced technologies: information [2, 3], numerical [4-6] and simulation modelling [7] as well as neural networks [8, 9]. In this case, it is possible to monitor the main characteristics of the investigated project in a safe manner and in great detail, which leads to quality improvement of the handed-over field materials and an expansion of the information set regarding the research object. In respect of geodetic support of the work, the use of advanced technologies allows combining geospatial data into a single set of reliable information used for taking engineering decisions and creating integrated models for safe operation of engineering constructions [10-12]. Non-contact methods of terrain surveying, such as laser scanning [13] and photogrammetric methods [14] including unmanned aerial vehicles [15-17] are actively applied. It should be noted that

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lately the observation methods with a satellite navigation system (GNSS) were applied to determine the altitude position of structural elements [18] for the efficient use of which it is necessary to have a quasigeoid height model or a height anomaly model for the construction site. It should be emphasized that in the process of designing and constructing the projects as well as in various urban development works, including land development, generation of a digital elevation model (DEM) based on the obtained data is an important stage for obtaining spatial information about the work site. Assessment of the accuracy and automation of DEM creation is a relevant task, since the quality and speed of construction allow increasing the productivity of all the related processes. Thus, the task of studying the accuracy of producing a local quasigeoid model and DEM arises in the absence of general recommendations for their construction [19, 20] and accuracy assessment [21, 22]. For example, articles [23-25] contain recommendations on the use of spatial interpolation methods [26, 27] for producing mathematical elevation models.

It should be noted that the construction of DEM is possible both with and without using a local quasigeoid model. However, this can lead to distortion of the information on topography [19, 25]. To provide the results of geodetic support, the normal height system is used [18, 28, 29]. In case of obtaining geodetic heights, the question of switching to it arises. Then, a local quasigeoid model is needed [30-32] for the creation of which it is important to have combined points [33, 34] (with known geodetic heights from the GNSS definitions and normal or orthometric heights from geometric levelling). Modelling of the surface of height anomalies [34] was caused primarily by land planning in Poland [28], Slovenia [31], coastal cities of the Red Sea [20], and Ethiopia [34].

The authors of [21, 35] noted that the production of such models in hard-to-reach mountainous areas is an important stage in accurate determination of heights using the GNSS equipment. Article [28] indicates that an accuracy of 5 mm was attained when creating a local model for the territory of Krakow. The production of such models is important for development of the regions [33]. Often, the problem of modelling the surface of height anomalies for an area project is solved. However, there are no studies aimed at creating a local quasigeoid model for an extensive linear project. No recommendations were drawn up concerning the required number of points or the method for assigning a sufficient number of points to create a model in which the accuracy of determining the heights would correspond to the accuracy indicated in work specifications at each stage of construction.

When determining the corrections to geodetic heights for further construction of DEM, it is necessary to choose the spatial interpolation method that will be used to construct a mathematical model of height anomalies [36]. DEM construction from data obtained by different survey methods [37] is actively investigated [35]. In this case, the results can be used as a spatial grid with regular spacing (GRID) or a point cloud [38]. Choice of the spatial interpolation method has a direct impact on the final accuracy of constructing the mathematical elevation model [39]. The most common method is the triangulation irregular network (TIN) [25]. However, in [40-42] the advisability of using other spatial interpolation methods is mentioned:

• The kriging method is used for constructing the topography of both the earth's, and the underwater surface [43, 44] with a correct definition of the variogram [45, 46] forming the spatial dependence.

• Radial basis function. The data on topography obtained by airborne laser scanning (ALS) are characterized by outliers which have a negative impact on the accuracy of digital elevation models. However, experimental results in the study [47] showed that the use of spline functions provides a more accurate model based on natural, rather than synthetically created data.

The obtained accuracy of constructions is affected by the density of the source geospatial data [48, 49] and the source of information [50]. The aim of the work is to assess the accuracy of creating mathematical elevation models and a quasigeoid based on spatial interpolation methods as well as to develop the initial provisions of the general methodology for creating a local quasigeoid model and a digital elevation model.

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Methods

Source data. *Construction of a local quasigeoid model*. Source data for the area project were 300 combined geodetic points at which geodetic and normal heights were determined. Of the total number of points at approximately the same spacing in different parts of the project, 10 % (30 points) were used as controls and did not participate in creation of the model. Root mean square errors (RMSE) of determining the normal heights of points did not exceed 3 mm. The area of the study project was 776.39 km^2.

When creating a local quasigeoid model for constructing a motorway in flat terrain, 28 points lying at different sides and at equal distances from the axis of the designed project in its different parts were selected from the total number of combined geodetic points. The limitation in the number of points is due to the existing plan-elevation datum. Similarly, to control the quality of construction, a part of points (10 %) was excluded from the process of creating models, which contradicted the principle of a uniform distribution of data and could also lead to collisions in the interpretation of the results of accuracy assessment due to local instability.

Construction of DEM. Source data were the results of airborne laser scanning of three large areas on the earth's surface with different types of topography (Table 1). Using the TerraScan software by Terrasolid Company and a point cloud macro, two point clouds were automatically classified, containing:

• key points used to construct the DEM;

• points on the earth’s surface that are not mandatory (key) for construction. At the same time, they contain spatial information about the earth’s surface, which can be used to assess the accuracy.

Then, the areas were visually assessed for the presence of points with non-characteristic deviations, which indicate an erroneous classification of individual points of the earth’s surface.

Table 1

Brief characteristic of plots

Plot	Average inclination angle, deg	Minimum height, m	Maximum height, m	Area, m^2
A	0.5	8.0	35.40	53,423
B	29.3	194.95	918.07	10,872
C	25.6	460.63	1,726.00	32,541

Plot A is a flat area (Fig.1, *a*) with minor local changes in surface curvature. Plot B (Fig.1, *b*) is a mountainous region with significant elevation differences and areas with major changes in surface curvature. Plot C is a mountainous area (Fig.1, *c*) with pronounced anthropogenic topography. Surface shows of disturbances and edges and slopes typical of open-pit mining are visible.

General description of the methodology for constructing models. Six methods of spatial interpolation were used. Constructions were performed in the Golden Software Surfer. It is proposed to create a local quasigeoid model:

• import of a set of geospatial data into the software;

• transition from a point cloud to GRID (spatial grid with regular spacing);

• construction of mathematical models by selected methods of spatial interpolation using different parameters;

• calculation of metrics for assessing the accuracy of constructions using control points;

• comparative analysis of metrics characterizing the accuracy of construction of mathematical models obtained using selected methods of spatial interpolation;

• definition of spatial interpolation method based on the values of selected metrics for constructing the mathematical model of similar plots.

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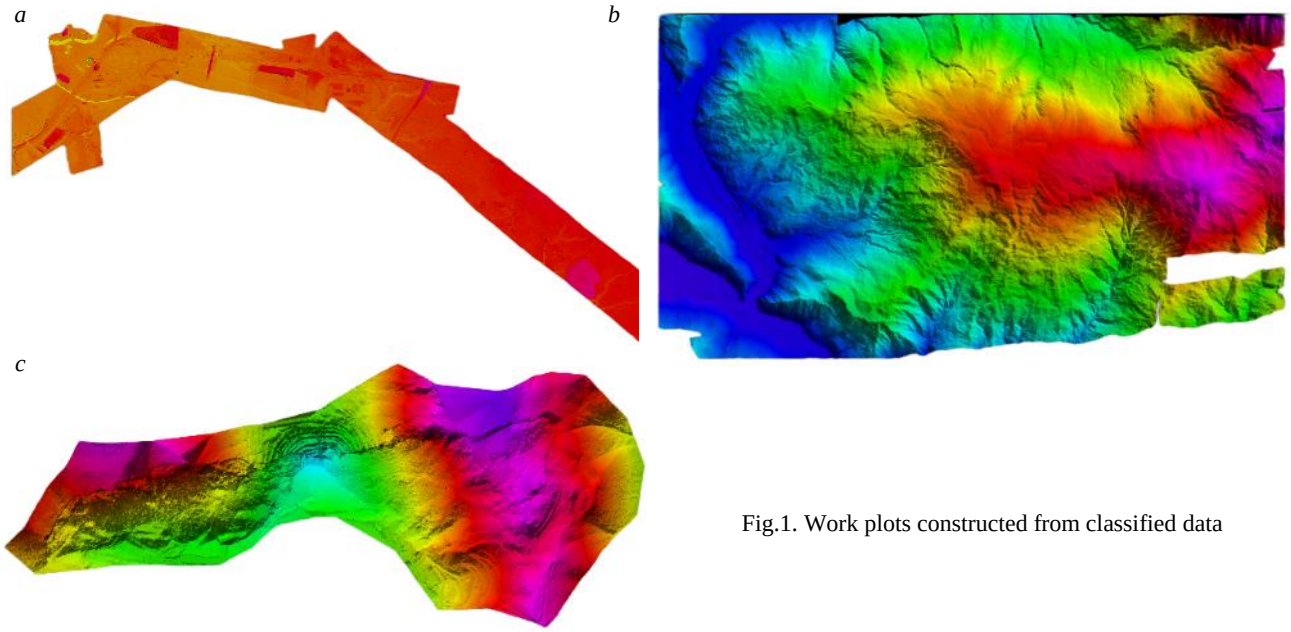


Fig.1. Work plots constructed from classified data

Creation of DEM:

• transformation of geodetic heights to normal ones in case of a large extent of the project site or according to the task set using the produced local quasigeoid model;

• division of a large area project into fragments 1,000×1,000 m;

• selection of a characteristic plot based on general morphometric characteristics from a set of obtained fragments;

• construction of DEM on a characteristic plot and drawing-up of practical recommendations for choosing the spatial interpolation method based on the accuracy assessment for the entire project site;

• creation of DEM for the entire project site using the selected spatial interpolation method.

Analysis of methods for constructing models. *Triangulation with linear interpolation*. In the Surfer program, a triangulation surface is constructed on the source data set. When a point with an unknown Z coordinate falls into the constructed plane limited by three points and determined by the equation

$$aX + bY + cZ + d = 0,$$

the coefficients are determined by expressions:

$$a = Y_1(Z_2 - Z_3) + Y_2(Z_3 - Z_1) + Y_3(Z_1 - Z_2);$$

$$b = Z_1(X_2 - X_3) + Z_2(X_3 - X_1) + Z_3(X_1 - X_2);$$

$$c = X_1(Y_2 - Y_3) + X_2(Y_3 - Y_1) + X_3(Y_1 - Y_2);$$

$$d = X_1(Y_2Z_3 - Y_3Z_2) + X_2(Y_3Z_1 - Y_1Z_3) + X_3(Y_1Z_2 - Y_2Z_1).$$

The unknown Z coordinate for a new point inserted into the surface is calculated from the formula

$$\hat{Z}_0 = \frac{-aX - bY - d}{c}.$$

Minimum curvature method. Using the least squares method, a mathematical surface is calculated that includes all the source data:

$$Z(X,Y) = a + bX + cY.$$

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Difference between the resulting construction values and points with the known coordinates is determined by the formula

$$e_i = (a + bX_i + cY_i) - Z_i.$$

At the next stage, the values of heights at nodes of the regular grid are calculated. The problem of solving a modified differential equation arises

$$(1-T_i)\nabla^2\left(\nabla^2 e\right)-(T_i)\nabla^2 e=0,$$

where T_i is the tension coefficient.

It is necessary to take the boundary conditions into account:

$$(1-T_b)\frac{\partial^2 e}{\partial n^2}+(T_b)\frac{\partial e}{\partial n}=0,\ \frac{\partial\left(\nabla^2 e\right)}{\partial n}=0,\ \frac{\partial^2 e}{\partial X\,\partial Y}=0,$$

where ∇^2 is the Laplace operator; n – boundary normal; T_b – boundary tension parameter.

Then, the final estimate of Z is determined by expression

$$\hat{Z}(X,Y)=a+bX+cY+\hat{e}(X,Y).$$

Using the minimum curvature method, surfaces are constructed with a change in two parameters: The Internal Tension (deflection amount) and The Boundary Tension (deflection value at edges). The higher the values of these parameters, the less pronounced the bending.

Nearest neighbour method. To determine the value of a new point added to the surface, the value of the nearest sample point is used

$$\hat{Z}_0 = Z_i,$$

where Z_i is the value of the nearest sample point.

In the process of creating models, different options for the search area of values were used in order to identify the impact of changes in the area on the accuracy of the digital model.

Natural neighbour. Definition of weights for calculating the values of coordinates at nodes of the regular grid is accomplished on the basis of proportional areas [40]:

$$\hat{Z}_0 = \sum_{i=1}^{N} w_{i0} Z_i\ ,$$

where w_{i0} is the weight of the i-th point (calculated using Voronoi diagrams).

When using such method, the heights of interpolated points will not go beyond the range of heights of the source data [41].

Radial basis function. To calculate the Z coordinate for the inserted point, the following expression is used

$$\hat{Z}_0 = \sum_{i=1}^{n} \lambda_i B(d_{i0}),$$

where d_{i0} is distance between the determined point and the known i-th point; λ, coefficient of the i-th point with known coordinates; B, radial basis function whose argument is d_{i0} distance.

The functions available in the Surfer were used as the compute core for the method in the investigation:

• Multiquadric

$$B(d)=\sqrt{d^2+R^2}\ ;$$

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• Inverse Multiquadric

$$B(d) = \frac{1}{\sqrt{d^2 + R^2}};$$

• Miltilog

$$B(d) = \log\left(d^2 + R^2\right);$$

• Thin Plate Spline

$$B(d) = \left(d^2 + R^2\right)\log\left(d^2 + R^2\right);$$

• Natural Cubic Spline

$$B(d) = \left(d^2 + R^2\right)^{3/2},$$

where d is distance from the point with an unknown Z value to the point with known spatial coordinates; R^2, smoothing parameter.

Kriging. The advantage of such method is the use of statistical models, which, among other things, allow making a forecast with an assessment of its accuracy. An important factor influencing the correlation coefficient is the distance between the initial points.

Z coordinate for a point added to the mathematical surface is determined from the formula

$$\hat{Z}_0 = \sum_{i=1}^{n} \lambda_i Z_i,$$

where n weights of λ_i is the solution of the kriging system

$$\sum_{i=1}^{n} \lambda_i C_{ij} = C_{oj}, \quad j = 1, ..., n.$$

The choice of spatial interpolation methods is determined by national and foreign experience of using them to construct digital elevation models and quasigeoids. In addition, the developers of the Surfer software indicate such methods for construction of digital elevation models.

Discussion of results

Construction of a local quasigeoid model of an area project. After the stage of model construction was completed, a comparison of methods was made on the basis of analysing the RMSE values of spatial interpolation and the RMSE of determining the height anomalies by control points (Table 2). It should be noted that the Gauss formula was used to calculate the RMSE of determining the height anomaly by control points. In the analytical examination of the assessment of the accuracy of constructions, the following features are highlighted:

• the least value of the interpolation RMSE was attained using the methods of triangulation with linear interpolation (0.003 m) and kriging (0.003 m);

• the least RMSE value for determining the heights by control points was obtained using the natural neighbour (0.004 m) and kriging (0.004 m) methods.

Table 2

Assessment of the accuracy of constructing a local quasigeoid model of an area project

Method	Number of points to create a model	Number of test points	RMSE of interpolation, m	RMSE of determining height anomalies from control points, m
Triangulation with linear interpretation	268	30	0.003	0.008
Minimum curvature	269	30	0.010	0.006
Nearest neighbour	265	30	0.014	0.006
Natural neighbour	265	30	0.007	0.004
Radial basis function (cubic spline)	263	30	0.007	0.006
Kriging	270	30	0.003	0.004

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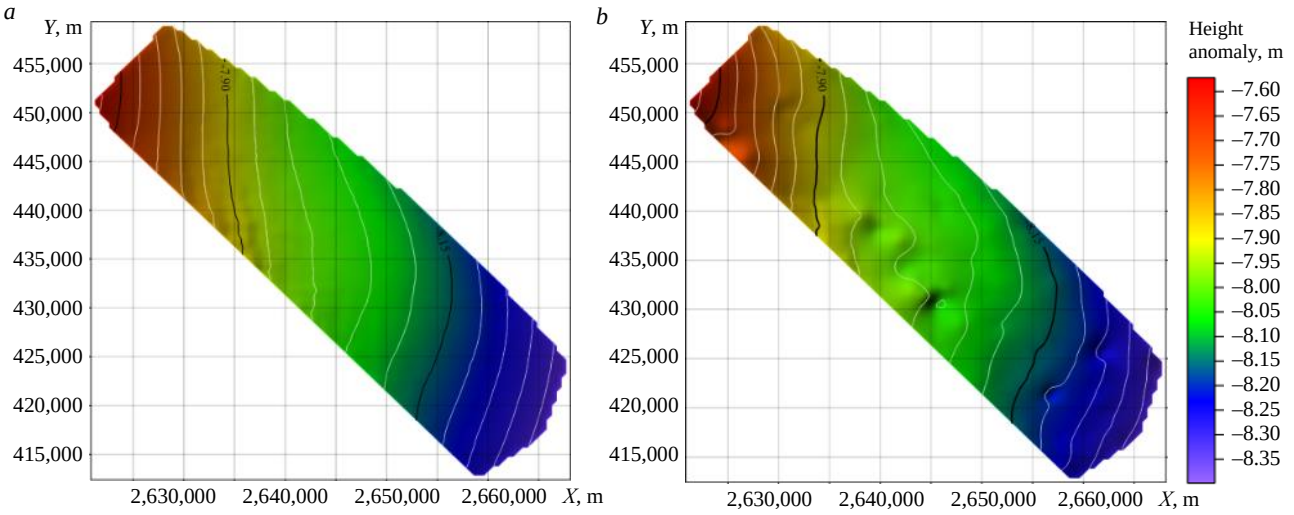


Fig.2. Local quasigeoid model constructed by triangulation methods with linear interpolation (*a)*; kriging (*b*)

Figure 2 shows the results of constructing local quasigeoid models a using triangulation with linear interpolation and kriging methods.

Construction of a local quasigeoid model of a linear project. To construct the model, data on geodetic and normal heights of points lying along the projected route were used. Similar constructions were made (Table 3). The least RMSE value for determining the heights from the model was obtained by kriging (0.006 m) and triangulation with linear interpolation (0.006 m). When assessing the accuracy of constructing a model based on control points, the least RMSE value for determining the heights was obtained using the kriging method (0.007 m). The kriging method on the estimated parameters is preferable for both projects. When determining the optimal number of initial points required constructing a local quasigeoid model, the points were equidistant from each other. Spacing between the neighbouring points was 5-6 km, and they lay on the same line. To assess the required number of points, it was decided to successively exclude the initial points and assess the accuracy of constructing the model based on control points. To build a model of 27 points, it was decided to leave 10 points, provided that they were uniformly distributed. The model was built using the kriging method, the RMSE of determining the heights of the model using the Gauss formula was 4 mm. The RMSE of building the model by this method was 7 mm. With further exclusion of points and building the model (with a 10 km spacing between points), the RMSE was 26 cm, which does not satisfy the required accuracy of construction. Thus, when designing the required number of points in a given area, the recommended spacing between the combined points is 5 km.

Table 3

Assessment of the accuracy of constructing a local quasigeoid model of a linear project

Method	Number of points to create a model	Number of test points	RMSE of interpolation, m	RMSE of determining height from control points, m
Triangulation with linear interpolation	27	10	0.006	0.009
Minimum curvature	28	9	0.01	0.009
Nearest neighbour	27	9	0.021	0.016
Natural neighbour	26	9	0.011	0.008
Radial basis function (cubic spline)	26	10	0.015	0.009
Kriging	28	10	0.006	0.007

Construction of DEM based on airborne laser scanning results. *Subdivision of areas into fragments.* Classified airborne laser scanning data were divided into fragments measuring 1,000×1,000 m. After this, the fragments that best characterize each plot were selected based on inclination angle and visual assessment.

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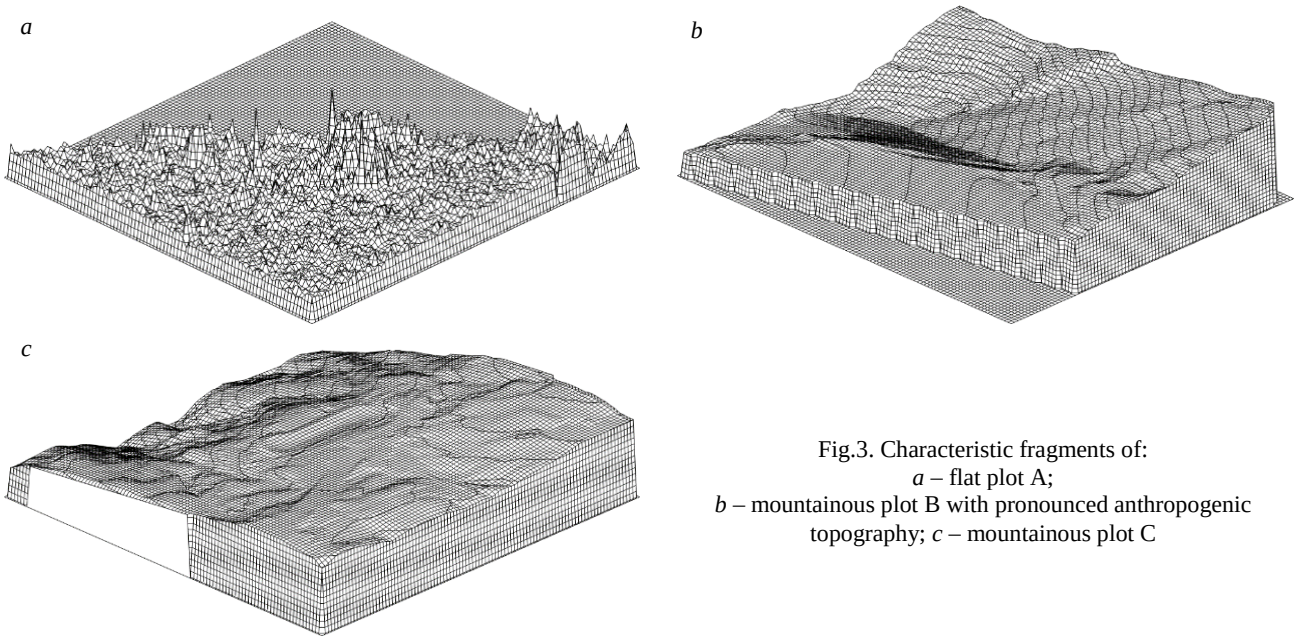


Fig.3. Characteristic fragments of:
a – flat plot A;
b – mountainous plot B with pronounced anthropogenic topography; *c* – mountainous plot C

Assessment of the accuracy of constructed models based on characteristic fragments using different spatial interpolation methods. To evaluate the accuracy, DEM were constructed using six spatial interpolation methods in the Surfer software. In total, 180 DEM were used, 60 models for each of three plots (Fig.3). This approach increases the speed of DEM construction for the entire project, since in the analysis of spatial interpolation methods on one characteristic fragment out of *N* fragments, the time for assessing the accuracy of model construction by the above methods decreases *N* fold provided that the hypothesis is true.

To analyse the spatial interpolation methods for accuracy, the deviation of a point on the earth's surface was calculated (two data sets), which can be considered as redundant measurements, since they were not involved in building digital elevation models. The choice of parameters "Percentage of points deviating by more than 0.33 m", "Percentage of points deviating by more than 1 m" is determined by the adopted contour interval for plots with flat topography. The first parameter is the percentage of points with deviation from the constructed surface exceeding 1/3 of contour interval; the second parameter is with deviation from the constructed topographic surface of at least the contour interval. Contour interval for the flat fragment *a* and plot A is taken to be 1 m (Table 4). Thus, the least value of parameters "RMSE of height determination" and "Percentage of points deviating by more than 0.33 m" was obtained using the methods of natural neighbour (0.06 m, 1.13 %), triangulation with linear interpolation (0.07 m, 1.50 %), and ordinary kriging with Power variogram (0.07 m, 1.82 %).

Table 4

Assessment of the accuracy of constructed models on flat fragment *a*

Method	Parameters	RMSE of height determination, m	Percentage of points deviating by more than 0.33 m	Percentage of points deviating by more than 1 m
Natural neighbour	Standard	0.06	1.13	0.00
Triangulation with linear interpolation	Standard	0.07	1.50	0.00
Kriging	Ordinary type Power variogram	0.07	1.82	0.04
Minimum curvature	Internal Tension: 0.75 Boundary Tension: 0.75	0.07	1.95	0.02
Radial basis function	Multilog function	0.07	2.18	0.05
Nearest neighbour	Standard	0.09	2.58	0.08

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DEM was constructed for a mountainous fragment with a pronounced anthropogenic factor. The choice of parameters "Percentage of points deviating by more than 1.66 m", "Percentage of points deviating by more than 5 m" is due to the adopted contour interval for the mountainous plot with anthropogenic topography. The first parameter is the percentage of points with deviation from the constructed surface exceeding 1/3 of contour interval; the second parameter is with deviation from the constructed DEM of at least the contour interval. In the study, the contour interval for the mountainous plot with anthropogenic topography is assumed to be 5 m (Table 5).

Table 5

Assessment of the accuracy of constructed DEM on a mountainous fragment with anthropogenic topography *b*

Method	Parameters	RMSE of height determination, m	Percentage of points deviating by more than 1.66 m	Percentage of points deviating by more than 5 m
Natural neighbour	Standard	0.88	14.24	1.59
Kriging	Quadric type Variogram for Gaussian (normal) distribution	0.89	15.21	1.64
Triangulation with linear interpolation	Standard	0.94	15.31	2.12
Radial basis function	Multiquadric function	0.92	16.06	1.92
Nearest neighbour	Standard	0.95	16.13	1.81
Minimum curvature	Internal Tension: 0.75 Boundary Tension: 0.75	1.21	22.60	3.20

The least values of the RMSE and the parameter "Percentage of points deviating by more than 1.66 m" were obtained using the natural neighbour method (0.88 m, 14.24 %), quadric kriging with a variogram component according to the Gaussian (normal) distribution (0.89 m, 15.21 %), and triangulation with linear interpolation (0.94 m, 15.31 %).

DEM was constructed on a mountainous plot. The choice of parameters "Percentage of points deviating by more than 1.66 m", "Percentage of points deviating by more than 5 m" is due to the adopted contour interval for the mountainous plot and is similar to fragment *b*. The accuracy assessment results for the mountainous fragment *c* are given in Table 6.

Table 6

Assessment of the accuracy of constructed digital models on mountainous fragment *c*

Method	Parameters	RMSE of height determination, m	Percentage of points deviating by more than 1.66 m	Percentage of points deviating by more than 5 m
Kriging	Quadric type Variogram with a spherical component	0.30	1.04	0.00
Natural neighbour	Standard	0.31	1.28	0.00
Radial basis function	Multiquadric function	0.32	1.29	0.00
Minimum curvature	Internal Tension: 0.75 Boundary Tension: 0.75	0.39	1.34	0.00
Nearest neighbour	Standard	0.35	1.47	0.00
Triangulation with linear interpolation	Standard	0.33	1.74	0.18

Based on a comprehensive assessment of the RMSE parameters for determining the height and parameter "Percentage of points deviating by more than 1.66 m", it is possible to distinguish the quadric kriging method with a spherical component of variogram (0.30 m, 1.04 %).

Checking the accuracy of DEM constructed using spatial interpolation methods for the entire project. Construction and accuracy assessment of digital elevation models created for the entire area of plots A, B and C were accomplished. Thus, the results obtained at the previous stage are aimed at getting

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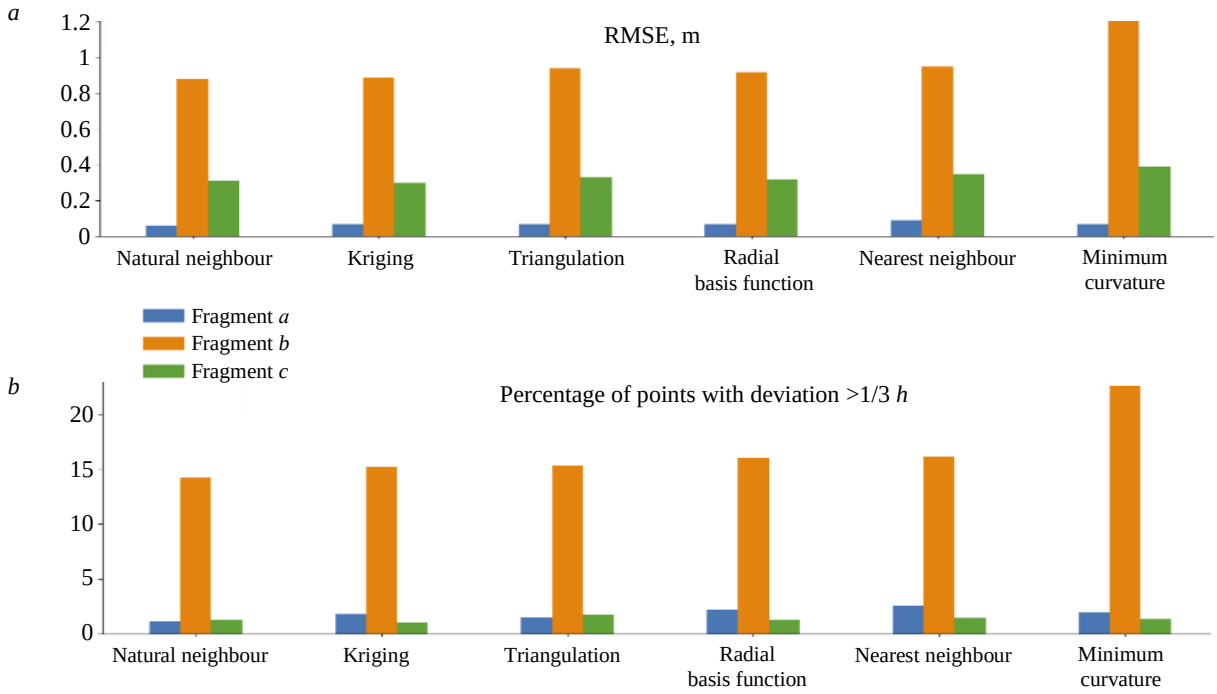


Fig.4. Assessment of the accuracy of methods for constructing DEM on the assigned fragments

a preliminary idea of the accuracy of DEM produced for each plot of the earth's surface, since differences in topography types clearly indicate the need to use a differentiated approach. Histogram with estimated parameters for fragments *a*, *b* and *c* is shown in Fig.4. A comprehensive assessment of the accuracy of constructed models for plots A, B and C is given in Table 7.

Table 7

Assessment of the accuracy of digital elevation models

Method	RMSE of height determination, m	Number of points	Percentage of points deviating by more than 1/3 *h* (m)	Percentage of points deviating by more than *h* (m)
Flat plot A				
Natural neighbour	0.11	53,288,238	5.84	0.84
Kriging	0.12	53,288,238	6.78	1.06
Triangulation with linear interpolation	0.12	53,288,238	6.93	1.09
Mountainous plot B with anthropogenic topography				
Kriging	0.55	4,556,172	6.63	0.77
Natural neighbour	0.57	4,556,172	7.17	0.80
Radial basis function	0.57	4,556,172	7.08	0.95
Mountainous plot C				
Kriging	0.29	17,476,315	0.83	0.01
Natural neighbour	0.29	17,476,315	0.91	0.01
Radial basis function	0.29	17,476,315	0.94	0.01

Methods of spatial interpolation according to the considered plots should be noted:

• Natural neighbour for plot A. Consistent with assessment of the accuracy of the characteristic fragment.

• Quadric kriging with a variogram component according to the Gaussian (normal) distribution and a natural neighbour for plot B. Partially consistent with assessment of the accuracy of the characteristic fragment. Differences in the comprehensive assessment appeared to be negligible, since the natural neighbour method showed greater resistance to the formation of outliers in the topographic

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surface. The kriging method slightly surpassed the natural neighbour method by other estimated parameters.

• Quadric kriging with a spherical variogram component for plot C. Consistent with a comprehensive assessment of the accuracy of the characteristic fragment.

Conclusion

In hard-to-reach regions, development or assessment of the integrity of the existing elevation datum is problematic. Creation of digital models will significantly reduce the production costs for repeated measurements of the investigated plots and search for the most productive solutions on choosing an algorithm for constructing a mathematical analogue with pre-assigned accuracy. Reducing the costs of constructing digital models will increase the availability of a single set of reliable geospatial information, which is used in designing, construction and further monitoring of buildings and engineering structures, in assessing the risk of landslides, flood monitoring and is the basis for research in Earth sciences based on background information.

Investigation of the accuracy of the local quasigeoid model showed:

• The least RMSE interpolation value for an area project was attained using triangulation with linear interpolation (0.003 m) and kriging (0.003 m). The least RMSE value for determining the heights by control points for an area project was obtained using the natural neighbour (0.004 m) and kriging (0.004 m) methods.

• The least RMSE for determining the heights from the model was shown by kriging (0.006 m) and triangulation with linear interpolation (0.006 m). When assessing the accuracy of constructing a model on control points, the least RMSE value for determining the heights was obtained using the kriging method (0.007 m).

• When constructing quasigeoid models, it is necessary to determine the number of starting points in the work area, which depends on anomality of the region. When designing the required number of points in a given area, the recommended spacing between combined points is 5 km.

Investigation of DEM construction accuracy showed:

• Construction of a digital elevation model applying the natural neighbour method on flat plot A of the earth's surface led to the least aggregate value of the estimated parameters: RMSE, "Percentage of points deviating by more than 0.33 m", and "Percentage of points deviating by more than 5 m".

• For DEM construction on mountainous plot B on the earth's surface with pronounced anthropogenic topography, the authors proposed the quadric kriging method with a variogram component according to the Gaussian (normal) distribution.

• For mountainous plot C, the authors identified the quadric kriging method with a spherical variogram component based on the least values of the estimated parameters.

• The approach to assessing the accuracy of construction based on the characteristic flat and mountainous fragments turned out to be efficient, since it was confirmed when constructing a DEM for the entire flat and mountainous project site. This is due to similar morphometric characteristics. On the mountainous fragment with anthropogenic topography such approach requires additional control by two spatial interpolation methods that are close to the optimal one in terms of the estimated parameters, which is accounted for by areas with a marked change in surface curvature.

All spatial interpolation methods resulted in deviations above the permissible ones for topographic surfaces on flat, mountainous plots with anthropogenic topography, and mountainous plots of the earth's surface with the least percentage deviations of 5.84, 6.63, and 0.83 %, respectively.

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