



Research article  
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## Comparison of the approaches to assessing the compressibility of the pore space

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**Abstract.** Integral and differential approaches to determining the volumetric compression of rocks caused by changes in the stress state are considered. Changes in the volume of the pore space of rocks are analyzed with an increase in its all-round compression. Estimation of changes in the compressibility coefficients of reservoirs due to the development of fields is an urgent problem, since the spread in the values of compressibility factors reduces the adequacy of estimates of changes in the physical properties and subsidence of the earth's surface of developed fields and underground gas storages. This parameter is key in assessing the geodynamic consequences of the long-term development of hydrocarbon deposits and the operation of underground gas storage facilities. Approaches to the assessment differ in the use of cumulative (integral) or local (differential) changes in porosity with a change in effective pressure. It is shown that the coefficient of volumetric compressibility of pores calculated by the integral approach significantly exceeds its value calculated by the differential approach, which is due to the accumulative nature of pore compression with an increase in effective pressure. It is shown that the differential approach more accurately determines the value of the pore compressibility coefficient, since it takes into account in more detail the features of the change in effective pressure.

**Keywords:** rock; porosity; compressibility of rocks; compressibility of pores; effective pressure; pore pressure; development of hydrocarbon deposits

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**Introduction.** The physical properties of reservoir rocks are divided into two main categories: porosity-permeability and deformation-strength. At the same time, the properties of rocks of the first category, primarily porosity and permeability, are basic in the preparation of projects for the development of hydrocarbon deposits and the calculation of their reserves [1-3]. Deformation-strength characteristics of rocks [4-6] (pore volume compressibility factor and tensile strength) are effectively used mainly to assess the deformation effects of long-term development of oil and gas fields and underground gas storages [7-9]. Changes in the stress-strain state cause the reaction of rocks, which is characterized by the coefficient of volume compressibility [10, 11]. The most well-known are tectonogenic changes in the stress-strain state of the subsoil, due to the development of oil and gas fields, which are accompanied by changes in the physical properties of reservoir rocks [8, 12]. Determination of fluid-saturated rocks volumetric compressibility is very important for the development of hydrocarbon deposits [10, 13]. Its role is that oil and gas can be squeezed into wells under the influence of pressure from overlying rocks during the development of deposits for depletion [14, 15]. Also, oil can be displaced into producing wells by special fluids injected into the reservoir. In this case, the actual volumetric compressibility of the pores will affect the production, among other factors. Accurate determination of the volumetric compressibility of the pore space is especially important in layers of weakly consolidated rocks, where a large value of the compressibility of the pores can lead to significant subsidence of the earth's surface [16, 17].



There are many ways to determine the compressibility of rocks [18, 19], but only in rare cases is it indicated under what conditions and at what pressures they were obtained [6, 20]. Large discrepancies in the values of compressibility factors significantly affect the adequacy of estimated changes in physical properties [21, 22] and subsidence of the earth's surface of developed hydrocarbon fields and underground gas storages [23, 24].

The dependence of the compressibility coefficient of the pore volume on the value of the effective pressure has a pronounced non-linear character [25, 26]. Therefore, it is important to know in what range of effective pressure changes the pore compressibility coefficient was estimated [27, 28]. This prompted the authors to consider the available approaches to assessing compressibility in more details and choose from them the most accurate and adequately reflecting changes in the stress state of rocks that do not exceed their elastic and/or strength limit.

The aim of the work is to analyze and compare the available approaches to assessing the compressibility of rocks and to select from them the most adequately reflecting changes in the effective pressure in the reservoir during the development of oil and gas fields and the operation of underground gas storage facilities.

**Methodology.** In compressible porous rocks, changes in either pore volume  $V_p$  or rock sample volume  $V_b$  are caused by changes in pore pressure or confining pressure [28-30]. The compressibility of the pore volume depends on changes in the compression pressure

$$C_{pc} = \frac{-1}{V_{p0}} \left( \frac{dV_p}{dP_c} \right)_{P_p} \quad (1)$$

or changes in pore pressure

$$C_{pp} = \frac{1}{V_{p0}} \left( \frac{dV_p}{dP_p} \right)_{P_c}; \quad (2)$$

in practice, the value of effective pressure is often used

$$P_{eff} = P_c - \alpha P_p, \quad (3)$$

where  $P_c$  – all-round compression pressure;  $P_p$  – pore (reservoir) fluid pressure (water, oil, gas);  $\alpha$  – the Biot coefficient, sometimes called the unloading coefficient, characterizes the part of the pore pressure, which counteracts the all-round pressure.

The  $\alpha$  coefficient is taken equal to one in the case of permeable rocks with a porosity of more than 3-5 % [31, 32]. Then  $C_{pc}$  and  $C_{pp}$  will be equal, since equal changes in the confining and pore pressures cause the same changes in the volume of the pore space, but their signs will be opposite. In this case, a decrease in the volume of the pore space will be caused either by an increase in all-round compression, or by a decrease in pore pressure [33]. In our case, the experiments were carried out under conditions when  $P_p = \text{const}$ , and the change in  $P_{eff}$  was achieved by increasing the all-round compression. In this case, the designation  $C_{pc} = C_p$  is appropriate. The pore compressibility coefficient exponentially depends on the effective pressure, which can be justified using the concepts of linear poroelasticity.

As is known, the compressibility coefficient of the pore space was determined by the formula

$$C_p = \left( \frac{-\Delta V_p}{V_{p0}} \right) / \Delta P_{eff}, \quad (4)$$

where  $\Delta V_p$  – change in the volume of the pore space (volume of pore fluid squeezed out of the sample),  $\text{cm}^3$ ;  $V_{p0}$  – the initial volume of the pore space of the sample,  $\text{cm}^3$ ;  $\Delta P_{eff}$  – changes in effective pressure, MPa.

Numerous experiments indicate that with increasing effective pressure, the pore compressibility coefficient decreases. In differential form, this corresponds to the equation



$$dC_p = -C_p b dP_{eff}, \quad (5)$$

where  $b$  – empirical coefficient that varies for different rocks and characterizes the intensity of changes in compressibility with changes in effective pressure,  $\text{MPa}^{-1}$ .

When integrating equation (5) in the range of changes in the compressibility coefficient from the initial value  $C_{p0}$  to the final  $C_p$  and from the initial effective pressure  $P_{eff0}$  to the final  $P_{eff}$ , we obtain the equality

$$\int_{C_{p0}}^{C_p} \frac{dC_p}{C_p} = -b \int_{P_{eff0}}^{P_{eff}} dP_{eff}. \quad (6)$$

When integrating and substituting limits, an exponential dependence  $C_p$  from  $\Delta P_{eff}$  is obtained:

$$\ln \frac{C_p}{C_{p0}} = -b(P_{eff} - P_{eff0}) = -b\Delta P_{eff} \quad (7)$$

or

$$C_p = C_{p0} e^{-b\Delta P_{eff}}. \quad (8)$$

Typical dependence of the coefficient of volumetric compressibility of the pore space on the change in effective pressure (Fig.1): increase in effective pressure from 0.1 to 37.0 MPa; trend approximation equation  $C_p = 0.0062e^{-0.0259P_{eff}}$ ; coefficient of determination  $R^2 = 0.98$ ; intensity of change in compressibility with change in effective pressure  $b = -0.0259$ . Naturally, with such a dependence of the compressibility coefficient on the effective pressure, the estimate of the coefficient will differ for different ranges of pressure change.

In this case, 34 sandy-argillaceous samples of the Vendian age of the Chayandinskoe field were used under conditions simulating reservoir conditions: porosity in atmospheric conditions is 3-15.6 %, gas permeability from  $2 \cdot 10^{-15}$  to  $100 \cdot 10^{-15} \text{ m}^2$ , the depth from which the core was taken is 1660-1860 m. The samples are mainly represented by coarse and fine-grained sandstones of quartz and quartz-feldspar composition with mixed quartz-regeneration, sulfate, clay-film and partially basal cement [34].

Depending on the chosen range of effective pressure variation, two approaches to estimating the pore compressibility coefficient are possible – integral and differential. This is especially true when the values of the pore compressibility coefficients are used to assess the anomalous deformations of the earth's surface of various oil and gas objects. So, for example, when assessing the subsidence of the earth's surface during the operation of oil fields, it is necessary to take into account the injection of fluid during the reservoir pressure maintenance procedure (RPM), which can restore the initial reservoir pressure and, accordingly, the effective pressure to the initial value after the fluid is withdrawn from the reservoir and reduce reservoir pressure, which can lead to a decrease in draw-downs, down to zero values. When injected into the reservoir, more complex effects are also possible, which go beyond the poroelastic concepts on which this work is based. When developing gas fields in the depletion mode, the increase in effective pressure occurs monotonously over a long period of time. The operation of underground gas storages (UGS) occurs in a cyclic manner, which leads to the need to take into account the intervals of alternating changes in effective pressure. Different modes of change

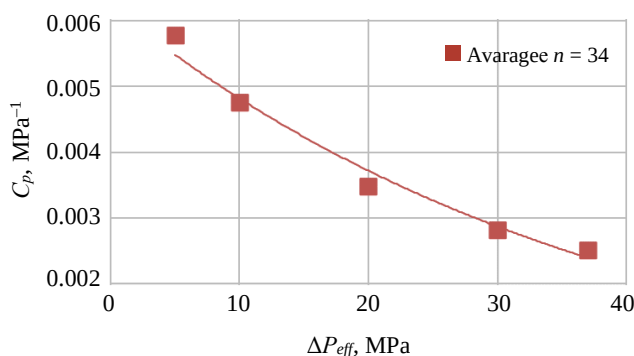


Fig.1. The change in the average value of the volumetric compressibility of the pore space for 34 samples, which is determined by the accumulated volume of liquid squeezed out of the samples – an integral approach



in effective pressure during the operation of oil and gas facilities lead to the need to use different approaches to determine the values of the pore compressibility coefficient.

*Integral* (averaged) approach implies an estimation of the compressibility factor for the entire range of effective pressure changes. In particular, the change in the coefficient  $\Delta C_p$  is determined in total for the entire interval of effective pressure change from  $P_{eff0}$  to  $P_{eff}$ . Usually, the value of the compressibility coefficient at an effective pressure close to atmospheric (0.1-2.0 MPa) is taken as the initial value, and for the final one – the value of the compressibility coefficient at the effective pressure for conditions simulating reservoir [31, 35], e.g. 37.0 MPa.

*Differential* approach involves estimating the change in the values of the compressibility coefficients in the selected interval of change in the effective pressure. It should be noted that the differential approach has two forms of representation: cumulative and discrete. In the first variant, the change in the pore volume is counted from the initial value  $V_{p0}$ , and its decrease is measured over the  $\Delta P_{eff}$  change intervals, which successively increase in the ranges, for example, 0.1-5.0; 0.1-10.0; 0.1-20.0 etc. up to the range of 0.1-37 MPa. In the second variant, the change in the pore volume is each time counted from the value of  $V_p$ , which corresponds to the value of the effective pressure from which the interval of its change begins, for example, 0.1-5.0; 5.0-10.0; 10.0-20.0; 20.0-30.0 and 30.0-37.0 MPa.

**Methodology and research object.** The methodology of experimental studies, described in detail in the work [33], allows you to directly measure the volume of the pore fluid  $\Delta V_p$  squeezed out of the sample and calculate not only changes in porosity, but also the volumetric deformation of the sample. Cylindrical sandstone samples with a diameter of 30 mm and a length of 30 mm were studied, which were subjected to all-round compression at a controlled pressure of the pore fluid. The use of a technique that makes it possible to measure the volume of the pore fluid squeezed out of the sample with an increase in the all-round compression of the sample made it possible to determine changes in the pore volume [34]. The  $\Delta V_p$  value in reservoir conditions is determined taking into account the fact that the compressibility coefficient of the solid rock matrix is several orders of magnitude greater than the compressibility coefficient of the pore space.

When carrying out petrophysical studies using this technique, the pressure and temperature of the pore fluid (formation water model) were maintained constant (13 MPa, 20 °C), only by increasing the all-round pressure to 37 MPa. Therefore, taking into account the coefficient of compressibility of the liquid on pressure and temperature was not required. Thus, based on formula (4), the coefficient of volumetric compressibility of the pore space was determined with a change in effective pressure.

In this case, to estimate the compressibility coefficient, an approach can be proposed based on parameters exclusively measured during the experiment: the volume of the pore space  $\Delta V_p$  and variations in the effective pressure  $\Delta P_{eff}$ . For this, formula (4) can be transformed into a differential form:

$$-dV_p = V_p C_p dP_{eff} . \quad (9)$$

Equation (9) implies the differential equation

$$\frac{dV_p}{V_p} dV = -C_p dP_{eff} . \quad (10)$$

Then, integrating equation (10) in the range of pore volume changes from the initial value  $V_{p0}$  to the final  $V_p$  and from the initial effective pressure  $P_{eff0}$  to the final  $P_{eff}$ , we obtain

$$\ln \frac{V_p}{V_{p0}} = -C_p (P_{eff} - P_{eff0}) = -C_p \Delta P_{eff} \quad (11)$$



or

$$V_p = V_{p0} e^{-C_p \Delta P_{eff}}. \quad (12)$$

When comparing formulas (7), (8) and (11), (12), it can be seen that formulas (11), (12) for determining the compressibility coefficient contain only quantities measured in this experiment, and formulas (7), (8) include the coefficient  $b$ , which must be determined from additional experiments.

**Discussion. Integral approach.** Determination of the pore volume compressibility coefficient in accordance with the integral approach can be performed graphically (Fig.2), using the linear dependence of changes in the logarithm of the pore volume on the effective pressure. From formula (11) it follows

$$\ln V_p = -C_p \Delta P_{eff} + \ln V_{p0}. \quad (13)$$

If we build a semi-logarithmic graph of the dependence of  $\ln(V_p)$  on the effective pressure, then we get the equation of an inclined straight line with a constant slope equal to  $C_p$  (Fig.2). The slope of the graph is the average value of the coefficient of volumetric compressibility of the pore space in accordance with the integral approach. The value of  $\ln(V_{p0})$  is determined from the condition  $\Delta P_{eff} = 0$ . In this case, the value of  $\ln(V_p)$  is dimensionless, which shows the degree to which ( $\sim 2.72$ ) a number equal to the current pore volume ( $V_p$ ) should be raised.

The obtained experimental dependence can be described with high reliability ( $R^2 = 0.94$ ) by the logarithmic equation (11):

$$\ln(V_{pi}) = -0.00249 P_{eff} + 0.631. \quad (14)$$

In this case, the average value of the coefficient is  $-0.00249 \text{ MPa}^{-1}$  (the slope of the graph in Fig.2) in the range of effective pressure changes of 0.1-37.0 MPa. And the free term of equation (14) 0.631 corresponds to the natural logarithm of the pore volume in the initial state.

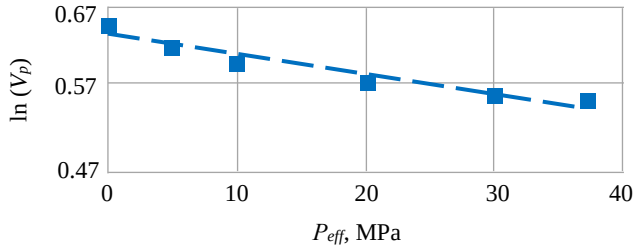


Fig.2. Pore volume change with increasing effective pressure on a semi-logarithmic scale

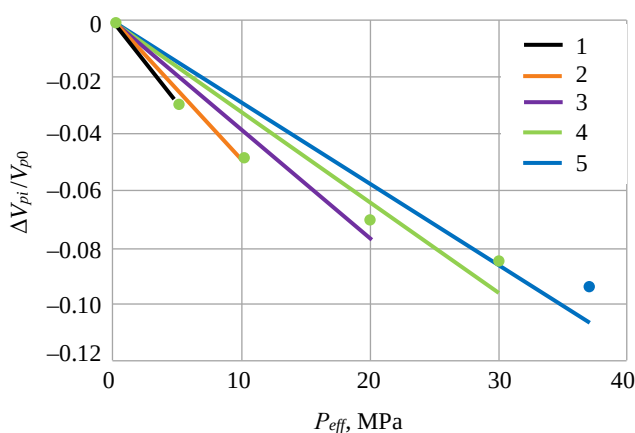


Fig.3. Graphical determination of the coefficient of volumetric compressibility of the pore space using the differential-accumulative approach with an increase in effective pressure  
1 – 0.1-5; 2 – 0.1-10; 3 – 0.1-20; 4 – 0.1-30; 5 – 0.1-37 MPa

Thus, the integral approach can be applied for graphical estimation of the average value of the volumetric compressibility of pores in the development of hydrocarbon fields only according to the data on the change in the volume of the pore space from the initial value  $V_{p0}$  to the current  $V_p$  when the effective pressure changes from the initial value to the corresponding reservoir conditions. However, the non-linear nature of the change in the compressibility coefficient from the effective pressure (see Fig.1) necessitates a detailed study of this dependence for various intervals of pressure change.

**Differential approach.** Based on the available experimental data, the entire range of effective pressure changes of 0.1-37.0 MPa was divided into local segments: 0.1-5.0; 0.1-10.0; 0.1-20.0; 0.1-30.0 and 0.1-37.0 MPa. At the same time, the values of the pore volume at an effective pressure of 0.1 MPa were taken as the initial values; the final values are 5.0; 10.0; 20.0; 30.0; 37.0 MPa. Figure 3 shows changes in pore deformation in the differential-accumulative approach with an increase in effective pressure.



Changes in pore deformation are zero at  $P_{eff} = 0.1$  MPa and increase with increasing effective pressure, approximated by linear dependences, the slope of which is the volumetric compressibility coefficient of pores  $C_{pi}$  (4). The  $C_p$  value is:  $-0.005653$ ;  $-0.004895$ ;  $-0.003795$ ;  $-0.003175$  and  $-0.002845$   $\text{MPa}^{-1}$  with changes in  $R_{eff}$ : 0.1-5.0; 0.1-10.0; 0.1-20.0; 0.1-30.0 and 0.1-37.0 MPa respectively.

Different intervals of effective pressure variation correspond to different values of the pore compressibility coefficient (Fig.3). This must be taken into account when assessing the final settlements of gas fields developed in the depletion mode [13, 14, 34].

The use of the differential-discrete approach is shown in Fig.4, which shows the linear dependences of changes in the volumetric deformation of pores on changes in effective pressure. In contrast to the differential-accumulative approach, the initial values are the pore volume at the beginning of each individual interval: 0.1; 5; 10; 25; 30; 37 MPa. Changes in pore deformation are equal to zero at  $P_{eff}$  equal to 0.1; 5; 10; 20 and 30 MPa, and increase with increasing effective pressure and are approximated by linear dependencies, the slope of which is the coefficient of pore volume compressibility  $C_{pi}$  (4).  $C_p$  values obtained:  $-0.005771$ ;  $-0.003880$ ;  $-0.002319$ ;  $-0.001669$ ,  $-0.001284$   $\text{MPa}^{-1}$  with the following changes in  $R_{eff}$ : 0.1-5.0; 5.0-10.0; 10.0-20.0; 20.0-30.0 and 30.0-37.0 MPa, respectively.

The volumetric compressibility coefficient of pores in this case is determined by the slope of the straight lines connecting the initial and final values of the volumetric deformation of pores with increasing effective pressure. In contrast to the differential-accumulative approach, for normalization (the ratio of changes in the pore volume to the initial value) changes in the volumetric deformation of pores  $\Delta V_p/V_p$ , the values of the pore volume at the initial value of the effective pressure of each stage of its growth are used. In Fig.4, both the general decrease in the value of the volumetric compressibility coefficient (from  $-0.00577$  to  $-0.00128$   $\text{MPa}^{-1}$ ) and the nonmonotonic nature of the decrease with increasing effective pressure are more clearly manifested.

The differential-discrete approach can be applied for various changes in the effective pressure, both with an increase and a decrease. However, it is necessary to have information about the volume of pores and their change at each stage of the development process. This is especially relevant when assessing anomalous deformations of the earth's surface during the operation of underground gas storage facilities, where a sign-alternating change in reservoir pressure is realized.

**Pore volume compressibility coefficient estimation comparisson.** Values comparison of the pore volume compressibility coefficients, which were obtained experimentally from the

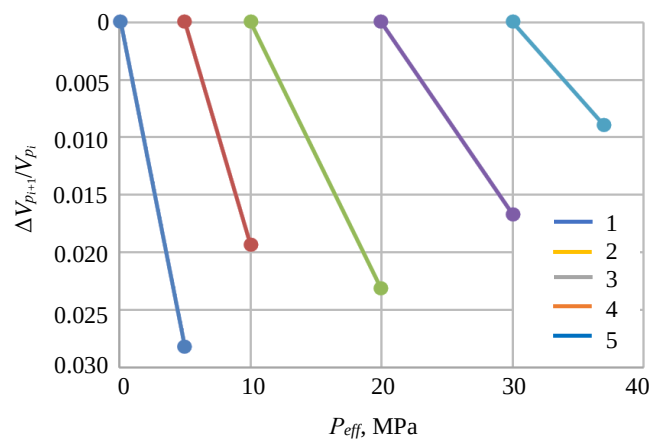


Fig.4. Graphical determination of the volumetric compressibility of the pore space coefficient using a differential-discrete approach for each interval of change in the effective pressure and the change in the volumetric deformation of the pores  
1 – 0.1-5; 2 – 5-10; 3 – 10-20; 4 – 20-30; 5 – 30-37 MPa

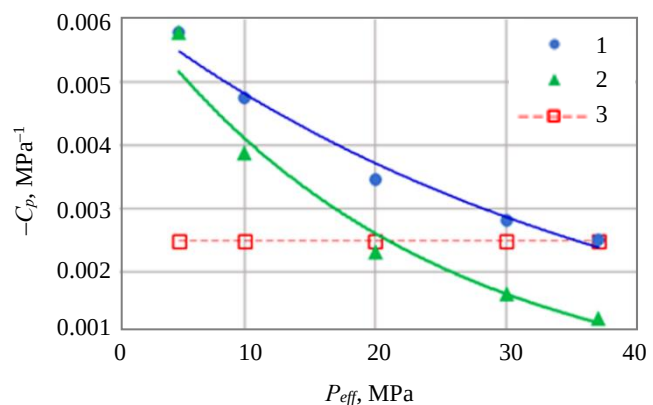


Fig.5. Change in pore volume compressibility factor according to 34 samples determined by increasing the effective pressure from 0.1 to 37 MPa

- 1 – differential-accumulative approach equation approximations  $dV_{pi}/V_{p0}$  (see Fig.3);
- 2 – differential-discrete  $dV_{p(i+1)}/V_{p_i}$   $n = 34$  (see Fig.4);
- 3 – integral, average over (13)  $n = 34$ ;  
 $C_p = -0.00249$   $\text{MPa}^{-1}$  (see Fig.2)



volumes of liquid squeezed out of the samples at each stage of effective pressure growth, and calculated using the integral and differential approaches, is shown in Fig.5.

Analyzing the Fig.1-5, it can be seen that the value of the volumetric compressibility coefficient, which is maximum at the initial (minimum) values of the effective pressure, decreases with its growth and can be approximated with a high degree of reliability ( $R^2 = 0.98$ ) by the exponential dependence (8) (see Fig.1). In this case, both integral and differential approaches can be used. And the average value when  $P_{eff}$  changes from 0.1 to 37.0 MPa with an integral approach can also be estimated using formula (13) (see Fig.2, 5).

Coefficient of volumetric compressibility of pores values are summarized in the Table.

Comparison of pore compressibility coefficients ( $C_p$ ,  $10^3 \cdot \text{MPa}^{-1}$ ) calculated using different approaches

| Approach                                                                                                  | $P_{eff}$ , MPa |        |        |        |        |        |
|-----------------------------------------------------------------------------------------------------------|-----------------|--------|--------|--------|--------|--------|
|                                                                                                           | 0.1             | 5.0    | 10.0   | 20.0   | 30.0   | 37.0   |
| Integral over changes in pore volume $\ln(V_p)$                                                           | –               | –2.495 | –2.495 | –2.495 | –2.495 | –2.495 |
| Differential-cumulative according to the accumulated volumetric deformation of the pores $dV_{pi}/V_{p0}$ | –               | –5.771 | –4.753 | –3.469 | –2.823 | –2.508 |
| Differential-discrete in steps of change $P_{eff}$ and volumetric deformation of pores $dV_{pi+1}/V_{pi}$ | –               | –5.771 | –3.880 | –2.319 | –1.669 | –1.284 |

At the initial effective pressure (5.0 MPa), the integral approach significantly underestimates the pore compressibility factor. In this case, the differential-accumulative and differential-discrete approaches coincide. It is obvious that the integral and differential-accumulative approaches significantly overestimate (up to 95.4 %) the value of the pore compressibility coefficient at effective pressures close to reservoir pressures (30-37 MPa) (Fig.5). It is also obvious that the use of the differential-discrete approach gives the values of the pore compressibility coefficient, which more adequately correspond to the stress state of the formation.

It is known that the effective pressure in the reservoir under development can vary in the range of  $\pm 10$ -20 MPa [8, 36]. At the same time, the differential (differential-discrete) approach proposed by the authors makes it possible to estimate the compressibility of the pore space even in the absence of laboratory studies of the core compressibility. It is easy to show that the change in normalized porosity is equal to the normalized change in pore volume:  $\Delta m/m_0 = \Delta V_p/V_{p0}$ . Then, using the values of changes in the porosity coefficient obtained during well logging within the framework of GIS control of the field development process or UGS operation, it is possible to estimate the pore volume compressibility coefficient for known changes in effective pressure. The differential approach makes it possible to more accurately assess the magnitude of possible subsidence of the earth's surface at various stages of field development.

Similarly, during the operation of underground storage facilities, to assess the pore compressibility factor, it is recommended to use a differential-discrete approach in order to more correctly (differentially) take into account changes in effective pressure occurring in the reservoir during gas injection and withdrawal periods. It is of fundamental importance that the values of the pore volume compressibility factors will be different for the period of gas injection and withdrawal even with the same change in reservoir (effective) pressure. This happens because the same (taking into account the sign) changes in the pore volume are normalized (divided) into different values of the pore volume corresponding to different values of the effective pressure. For example, with an increase in effective pressure from 10 to 20 MPa, the change in pore volume  $\Delta V_{p0}$  is divided by the value of the pore volume  $V_{p0}$ , which corresponds to a value of 10 MPa. When the effective pressure decreases from 20 to 10 MPa, the identical change in the pore volume  $\Delta V_p$  (in this case, the increase) is normalized to the value  $V_{p0}$ , which corresponds to the value of the effective pressure of 20 MPa. Naturally, in the second case, the compressibility factor will be greater than in the first, since the denominator of the



fraction  $\Delta V_p/V_{p0}$  in the first case will be greater than in the second. This must be taken into account in a detailed analysis of the results of geodynamic monitoring of underground gas storage facilities.

The results of the experimental data discussion make it possible to recommend laboratory differential approaches (cumulative and discrete) for the practical determination of the pore volume compressibility coefficient, since other external factors and uncertainties in pressure values that are present in field determinations during field development are excluded. The GIS-control method using changes in porosity can be recommended as a refinement method aimed at monitoring changes in the volumetric compressibility of the pore space and confirming laboratory results. For a detailed analysis of changes in the stress state of underground gas storage facilities, a differential-discrete approach to estimating the pore volume compressibility coefficient can be recommended, as it allows taking into account alternating changes in reservoir pressure.

Consistently taking into account the differences in compressibility coefficients also makes it possible to study such an important empirical fact as weak subsidence of the earth's surface in long-term developed gas fields in the depletion mode, which was noted based on the results of long-term monitoring at a field in Turkmenistan [8, 37]. To explain it, a genetic model was used for the formation of subsidence in the territory of deposits, which used the relationship between relative changes in the pore compressibility coefficient and relative changes in the volumetric modulus  $K$  of the host medium in an environment of permanent (quasi-static) stresses that formed the structure of the anticlinal uplift over geological time, to which the deposit is assigned. These forces (stresses) contribute to the uplift of the earth's surface, compensating for subsidence, under conditions of a drop in the effective stiffness (volumetric modulus) of the reservoir. It is known from geomechanics that the compressibility coefficient is inversely proportional to  $K$  [6, 8]. Relative (normalized) changes in the compressibility coefficient  $C_p$  are related to relative changes in the bulk modulus by the following formula:

$$\frac{\Delta C_p}{C_p} = -\frac{\Delta K}{K}. \quad (15)$$

The minus sign in equation (15) means that as the compressibility coefficient increases, the value of the bulk modulus decreases, and vice versa. Relation (15) makes it possible to make the transition from petrophysical models to models of poroelastic heterogeneity, on the basis of which it is possible to evaluate the change in the stress-strain state of the developed reservoir (layers) of oil and gas fields and underground gas storages.

The scientific novelty of the work lies in the analytical substantiation of the exponential dependence of the pore compressibility coefficient on the effective pressure, which was previously assumed in [34] and confirmed by the results of experimental studies on rock samples.

**Conclusion.** Comparison of the integral and differential approaches to determining the coefficient of volumetric compressibility of the rock samples pore space emphasizes the damping nature of its decrease with an increase in effective pressure, which can be approximated with a high degree of reliability by an exponential dependence, which has been proven using the basic relations of geomechanics of poroelastic media. It was found that the coefficient of pore volume compressibility calculated by the integral approach is significantly higher (up to 130 %) than the coefficient calculated using the differential approach. This difference is due to the cumulative nature of the decrease in pore volume with an increase in effective pressure. Obviously, the differential approach makes it possible to more accurately determine the compressibility of the pore space due to a more appropriate consideration of changes in the effective pressure in the reservoir and can be recommended for assessing the dynamics of the physical properties of rocks in the process of developing oil and gas fields and operation of underground storage facilities, as well as for studying anomalous deformations of the earth's surface within these objects.



## REFERENCES

1. Blinova E.Yu., Indrupsky I.M., Zakirov E.S., Kovalenko K.V. Accounting of a reservoir's compressibility heterogeneity while building hydrodynamic models of productive layers. *Geology, geophysics and development of oil and gas fields*. 2012. N 12, p. 32-35 (in Russian).
2. Gabisia B.C. Characteristics and influence factors for oil displacement efficiency determination in low-permeability and complex reservoir simulation. *Oil Industry Journal*. 2017. N 7, p. 50-53 (in Russian). DOI: 10.24887/0028-2448-2017-7-50-53
3. Galkin S.V., Krivoshchekov S.N., Kozyrev N.D. et al. Accounting of geomechanical layer properties in multi-layer oil field development. *Journal of Mining Institute*. 2020. Vol. 244, p. 408-417. DOI: 10.31897/PMI.2020.4.3
4. Kashnikov Yu.A., Shustov D.V., Kukhtinskii A.E., Kondratiev S.A. Geomechanical properties of the terrigenous reservoirs in the oil fields of Western Ural. *Oil Industry Journal*. 2017. N 4, p. 32-35. DOI: 10.24887/0028-2448-2017-4-32-35
5. Kashnikov Yu.A., Ashikhmin S.G. Rock mechanics in petroleum industry. Moscow: Izd-vo "Gornaya kniga", 2019, p. 552 (in Russian).
6. Fjær E. Relations between static and dynamic moduli of sedimentary rocks. *Geophysical Prospecting*. 2019. Vol. 67. Iss. 1, p. 128-139. DOI: 10.1111/1365-2478.12711
7. Kashnikov Yu.A., Ashikhmin S.G., Shustov D.V. et al. Increasing the efficiency of hydrocarbon field development based on integrated geomechanical studies. *Neftyanoe khozyaistvo*. 2019. N 3, p. 66-69 (in Russian). DOI: 10.24887/0028-2448-2019-3-66-69
8. Kuzmin Yu.O. Deformational Consequences of Oil and Gas Field Development. *Geophysical Processes and Biosphere*. 2021. Vol. 20. N 4, p. 103-121 (in Russian). DOI: 10.2d455/GPB2021.4-7
9. Kuzmin Yu.O. Modern volumetric deformations of fault zones. *Fizika Zemli*. 2022. N 4, p. 3-18 (in Russian). DOI: 10.31857/S0002333722040068
10. Sharifi J., Saberi M.R. Quantitative Evolution Fracture Porosity a Carbonate Reservoir Using Analytical Method. 83rd EAGE Annual Conference, 6-9 June 2022. DOI: 10.3997/2214-4609.202210135
11. Zhukov V.S., Kuzmin D.K., Kuzmin Yu.O., Pleshkov I.V. Comparison of forecast estimates of seabed subsidence of the Yuzhno-Kirinskoye field. IV National Scientific Conference with Foreign Participants: Geodynamical Processes and Natural Hazards (4th GeoProNH 2021), 06-10 September 2021, Yuzhno-Sakhalinsk, Russian Federation. IOP Conference Series: Earth and Environmental Science. 2021. Vol. 946. N 012019. DOI: 10.1088/1755-1315/946/1/012019
12. Zhukov V.S., Semenov Ye.O., Kuzmin Yu.O. Dynamics of physical properties of reservoirs at development of oil and gas fields. *Vesti Gazovoy Nauki*. 2018. N 5 (37), p. 82-99 (in Russian).
13. Zhukov V.S., Ivanov P.Yu. Change in the physical properties of the reservoir as a result of an increase in effective pressure during field development (modeling on the example of the Yuzhno-Kirinskoye field). *Vesti Gazovoy Nauki*. 2015. №4 (24), p. 144-148 (in Russian).
14. Sharifi J., Saberi M.R., Javaherian A., Moghaddas N.H. Investigation of static and dynamic bulk moduli in a carbonate field. *Exploration Geophysics*. 2021. Vol. 52. Iss. 1, p. 16-41. DOI: 10.1080/08123985.2020.1756693
15. Mavko G., Mukerji T., Dvorkin J. The Rock Physics Handbook, Second Edition: Tools for Seismic Analysis of Porous Media. New York: Cambridge University Press, 2009, p. 524.
16. Ranjbar A. Hassani H., Shahriar K. 3D geomechanical modeling and estimating the compaction and subsidence of Fahlian reservoir formation (X-field in SW of Iran). *Arabian Journal of Geosciences*. 2017. Vol. 10. N 116, p. 1-12. DOI: 10.1007/s12517-017-2906-3
17. Schutjens P.M.T.M., Hanssen T.H., Hettema M.H.H. et al. Compaction-induced Porosity Permeability Reduction in Sandstone Reservoirs: Data and Model for Elasticity-Dominated Deformation. *SPE Reservoir Evaluation & Engineering*. 2004. Vol. 7. Iss. 3, p. 202-216. N SPE-88441-PA. DOI: 10.2118/88441-PA
18. Vinogradov K.E., Pustoshkin R.V., Rodionov S.P. Features of accounting the hysteresis of permeability and compressibility of the pore space of low-permeability reservoirs in hydrodynamic modeling. *Geologiya, geofizika i razrabotka neftnykh i gazovykh mestorozhdenii*. 2021. N 11 (359), p. 35-38. DOI: 10.33285/2413-5011-2021-11(359)-35-38
19. Panteleev I.A., Lyakhovsky V., Mubassarova V.A. et al. Tensor compaction of porous rocks: theory and experimental verification. *Journal of Mining Institute*. 2022. Vol. 254, p. 234-243. DOI: 10.31897/PMI.2022.30
20. Hall H.N. Compressibility of Reservoir Rocks. *Journal of Petroleum Technology*. 2013. Vol. 5. Iss. 01, p. 17-19. SPE-953309-G. DOI: 10.2118/953309-G
21. Ghasemi M.F., Bayuk I.O. Bounds for Pore Space Parameters of Petroelastic Models of Carbonate Rocks. *Izvestiya, Physics of the Solid Earth*. 2020. N 2, p. 69-88 (in Russian). DOI: 10.31857/S0002333720020039
22. Petrakov D.G., Penkov G.M., Zolotukhin A.B. Experimental study on the effect of rock pressure on sandstone permeability. *Journal of Mining Institute*. 2022. Vol. 254, p. 244-251. DOI: 10.31897/PMI.2022.24
23. Nagel N. Compaction and subsidence issues within the petroleum industry: From Wilmington to Ekofisk and beyond. *Physics and Chemistry of the Earth, Part A: Solid Earth and Geodesy*. 2001. Vol. 26. Iss. 1-2, p. 3-14. DOI: 10.1016/S1464-1895(01)00015-1
24. Chin L., Nagel N. Modeling of Subsidence and Reservoir Compaction under Waterflood Operations. *International Journal of Geomechanics*. 2004. Vol. 4. Iss. 1, p. 28-34. DOI: 10.1061/(ASCE)1532-3641(2004)4:1(28)
25. Baud P., Teng-fong Wong, Wei Zhu. Effects of porosity and crack density on the compressive strength of rocks. *International Journal of Rock Mechanics and Mining Sciences*. 2014. Vol. 67, p. 202-211. DOI: 10.1016/j.ijrmms.2013.08.031
26. Shimin Liu, Harpalani S. Compressibility of sorptive porous media: Part 1. Background and theory. *AAPG Bulletin*. 2014. Vol. 98. N 9, p. 1761-1772. DOI: 10.1306/03241413133
27. Jun He, Kegang Ling, Peng Pei, Xiao Ni. Calculation of rock compressibility by using the characteristics of downstream pressure change in permeability experiment. *Journal of Petroleum Science and Engineering*. 2016. Vol. 143, p. 121-127. DOI: 10.1016/j.petrol.2016.02.030



28. Shihuai Zhang, Shunchuan Wu, Guang Zhang. Strength and deformability of a low-porosity sandstone under true triaxial compression conditions. *International Journal of Rock Mechanics & Mining Sciences*. 2020. Vol. 127, p. 1-13. DOI: 10.1016/j.ijrmms.2019.104204
29. David E.C., Zimmerman R.W. Compressibility and shear compliance of spheroidal pores: Exact derivation via the Eshelby tensor, and asymptotic expressions in limiting cases. *International Journal of Solids and Structures*. 2011. Vol. 48. Iss. 5, p. 680-686. DOI: 10.1016/j.ijsolstr.2010.11.001
30. Zimmerman R.W. Pore Volume and Porosity Changes under Uniaxial Strain Conditions. *Transport in Porous Media*. 2017. Vol. 119, p. 481-498. DOI: 10.1007/s11242-017-0894-0
31. Mao Bai, Xipu Shen, Gang Li. Alternative Method to Determine Pore Volume Compressibility Attributable to Production-Induced Reservoir Compaction. International Oil & Gas Conference and Exhibition, 8-10 June 2010, Beijing, China, p. 1-12. SPE-130212-MS. DOI: 10.2118/130212-MS
32. Selvadurai A.P.S., Suvorov A.P. The influence of the pore shape on the bulk modulus and the Biot coefficient of fluid-saturated porous rocks. *Scientific Reports*. 2020. Vol. 10. N 18959. DOI: 10.1038/s41598-020-75979-6
33. Zhukov V.S., Kuzmin Yu.O. Experimental evaluation of compressibility coefficients for fractures and intergranular pores of an oil and gas reservoir. *Journal of Mining Institute*. 2021. Vol. 251, p. 658-666. DOI: 10.31897/PMI.2021.5.5
34. Zhukov V.S., Kuzmin Y.O. The influence of fracturing of the rocks and model materials on p-wave propagation velocity: experimental studies. *Izvestiya, Physics of the Solid Earth*. 2020. Vol. 56. N 4, p. 470-480 (in Russian). DOI: 10.31857/S0002333720040109
35. Jia-Jyun Dong, Jui-Yu Hsu, Wen-Jie Wu et al. Stress-dependence of the permeability and porosity of sandstone and shale from TCDP Hole-A. *International Journal of Rock Mechanics and Mining Sciences*. 2010. Vol. 47. Iss. 7, p. 1141-1157. DOI: 10.1016/j.ijrmms.2010.06.019
36. Gumrah F., Aliyev A., Guliyeva C., Ozavc O. Determining Reservoir Characteristics and Drive Mechanisms for an Oil Reservoir. *SOCAR Proceedings*. 2012. Iss. 4, p. 6-19. DOI: 10.5510/OGP20120400129
37. Izyumov S.F., Kuzmin Y.O. Study of the recent geodynamic processes in the Kopet-dag region. *Izvestiya, Physics of the Solid Earth*. 2014. Vol. 50. N 6, p. 719-731. DOI: 10.7868/S0002333714060015

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