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Optimization of geometrical parameters of the hydro-cyclone inertial Venturi separator

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The usage of nanosized particles as modifying agents opens new possibilities in the creation of materials with unique properties. The effective qualitative improvement of Russia's GDP structure is based on the recycling of technogenic mineral formations (TMF) and the production of high-tech products. Numerous studies have shown that the efficiency of this process is limited by high requirements to the fractional composition, median size, and dispersion of TMF particles, as well as imperfection of equipment and technology and their classification. The strict classification requirements must be taken into account, when developing separation methods for the dispersion of the median sizes of TMF microparticles under the conditions of the probabilistic distribution of the physical and mechanical parameters of the feed. The studies covered in the article are based on the provision on a significantly greater influence of inertial forces on the trajectory of a hydrodynamically unsteady motion of the dispersed «a microparticle – a drop of liquid» system during the hydro-cyclone separation with respect to the aerodynamic forces of their movement in a fluidized bed. The paper shows that within the range of kinetic energy of the translational motion of liquid droplets, which overcomes the aerodynamic barrier of coagulation of hydrophobic TMF particles, the minimum diameter of absorbed microparticles during hydro-cyclone coagulation depends only on the magnitude of the angular velocity of rotation of the liquid droplets.

We obtained the equations for the Euler and Reynolds criteria, their average values, and the relaxation time of liquid droplets with integrated micro and nanoparticles of TMF, depending on their median size during hydro-cyclone separation. The developed mathematical model of inertial hydro-cyclone separation of finely dispersed TMF allows determining the optimal geometric parameters and energy characteristics of the Venturi separator, its aerator, and the position of the receiving tanks.

The experimental results confirmed the possibility of classifying finely dispersed wastes of mining and metallurgical production in the range of median sizes $(0.5\text{-}5)\cdot10^{-6}$ m by fractions with a dispersion of not more than 20 %.

Key words: recycling; separation; cyclone heterocoagulation; Reynold and Euler criteria; hydrophobicity; median size; dispersion; Venturi pipe; inertia forces

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Introduction. Promotion of competitiveness of the mining and metallurgical companies (MMC) of the Russian Federation on the global market is impossible without promoting the improvement of existing sites and commissioning of new ones based on the implementation of advanced R&D tools and introduction of modern high-tech equipment for the production of goods and materials with unique functional properties. To a large extent, this strategy implies technological changes in the preparation and sorting of raw materials, which significantly affects the quality of the final product.

The most effective strategy for improving Russia's GDP structure is the introduction of recycling technology, i.e., the use of technogenic mineral formations (TMF) in the production of high-tech products.

The use of nanosized particles as modifying agents and individual materials opens new possibilities for using known materials. The application of nano-powders for the implementation of new complex functional properties has no alternative when creating high heat dispersion-strengthened composite materials [2].

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Returned nano alumina dust becomes a recycle ballast with the mass fraction up to 7-14 % of the total amount of produced alumina. Given that the annual production of alumina in the Russian Federation is estimated at 11.5 million tons, the mass of recycled alumina dust is a significant amount. Alumina dust can be used as alloying elements in the production of refractory materials and the electronics industry [3, 11, 16].

Numerous studies have shown that the efficiency of recycling is limited by high requirements to the fractional composition, median size, and dispersion of TMF particles. In most cases, the required size of micro and nanoparticles and their dispersion are in the following ranges: $d_p = (0.1\text{-}6)\cdot 10^{-6}$ m; $3\sigma = 0.2d_m$. One of the constraining factors for increasing the efficiency of recycling of the finely dispersed solid wastes is the high energy intensity and imperfection of classification equipment and technology [5, 10, 12].

A significant drawback of the existing classification tools is the lack of efficiency of the formation of a narrow range of trapped fractions for micro and nanosized particles [13-15].

The stringent classification requirements to the dispersion of the median sizes of finely divided TMFs necessitate the search for methods and tools that can meet these requirements under the conditions of the probabilistic distribution of the physical-mechanical, geometric, and kinematic properties of nanoparticles.

Formulation of the problem. To supply the production of materials with unique properties with high-quality raw materials, we need a technology in which the external control over the classification by the median-size dispersion will be self-similar, i.e., irrespective of the probabilistic characteristics of the TMF physical-mechanical properties.

The paper is devoted to the development of the mathematical model of the hydro-cyclone separation in a fluid bed of micro and nanoparticles of TMF and a device for its implementation, which is a Venturi hydro-cyclone separator.

The task is to develop a method for the separation of TMF particles with a size of $(0.5\text{-}6)\cdot 10^{-6}$ m with a dispersion of median sizes of ± 5 % and the efficiency of the content of a given fraction in the total mass of sampling along the Tromp curve 95-97 %, as well as the creation of hydro-cyclone separators, in which the separation of particles in a given size range is guaranteed by their coagulation by rotating drops of liquid. The sorting should be ensured by the parameters of the Venturi separator, aerator, and hydro-cyclone nozzles based on the proposed mathematical model, as well as the energy characteristics of rotating drops and a fluid bed of finely dispersed TMF.

Methodology and object of research. Inertial hydro-cyclone separation is the process of separating particles of a given median size and dispersion from TMF in a fluidized bed state by forming their motion paths to a receiving tank under the influence of inertial forces of an unsteady process of hydro-cyclone interaction with rotating liquid drops over a period not exceeding the relaxation time.

To establish the relationship between the energy of a dispersed system «a drop of liquid – a TMF microparticle» and its vertical energy flow in a fluidized bed and the geometric parameters of the Venturi separator during inertia hydro-cyclone separation in the relaxation phase, it is necessary to obtain the equations of particle motion depending on the forces acting on them.

The used method of additive aerohydrodynamic calculation of hydro-cyclone separation is distinguished by independent correction of the influence of the particle velocity in the fluidized bed and the speed of the unsteady inertia hydro-cyclone motion of the dispersed system «a drop of liquid – a TMF microparticle». This approach allows us to study the influence of the kinematic and energy parameters of rotating liquid droplets on the particle trajectory and to determine the dependence of fractional separation efficiency on the geometric and energy parameters of the separator.

Technically, the problem is solved in two stages. In the first stage, we study the hydrodynamic interaction of liquid droplets and TMF particles at large Reynolds numbers in the steady-state mode using the basic principles of similarity theories to obtain the dependence of the drag coefficient k_i

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on the Reynolds and Euler criteria. We used the equations of steady-state Stokes motion in classical coagulation in the form of the Boussinesq equation to create the valid mathematical model of the hydro-cyclone separation [1, 7].

In the second stage, we established the main laws of change for the indicators of similarity obtained in the first stage of their correlation with relaxation time τ_i and resistance coefficient k_i under conditions of unsteady motion. Consideration of the transient motion of a liquid droplet, followed by averaging the aero-hydrodynamic parameters of its motion over the relaxation time, and taking into account the regularity of the transient over-Stokes motion, allows us to solve the problem of averaging the resistance coefficient k_i to high precision [7, 8]. Thus, the dynamically active section of hydro-cyclone coagulation in the Venturi separator, characterized by an unsteady over-Stokes flow mode in which the drag force significantly depends on the Reynolds and Euler criteria, can be represented by a mathematical model with averaged values of the criteria describing this process.

In paper [6] authors proposed the graphic-analytical model of orthokinetic hydro-cyclone heterocoagulation based on the well-known model of kinetic heterocoagulation of dust particles by a liquid drop at an angular velocity of liquid $\omega_l = 0$, it presents a system of levels describing the physical process of absorption of solid particles by rotating drops of liquid with the participation of additional cyclone energy.

Using the proposed graphical analytical model of inertial hydro-cyclone orthokinetic heterocoagulation, we obtained the equation for the minimum diameter of the absorbed solid component in the liquid-solid system for rotation of a liquid droplet with an angular velocity ω_l and effective contact angle in the contact zone of the liquid and solid phases, taking into account the additional energy due to the rotation of a drop of liquid.

For absolutely hydrophobic particles, such as TMF microparticles with a diameter of less than $6 \cdot 10^{-6}$ m, the hypothesis of direct correlation of the minimum diameter of completely absorbed particles with the angular velocity of rotation of liquid droplets during hydro-cyclone inertial kinematic heterocoagulation is confirmed mathematically and experimentally. Moreover, beyond the threshold of the aerodynamic barrier, the physical-mechanical and chemical properties of particles do not affect the wetting process in the range of diameters of absorbed particles corresponding to the cyclone and classical coagulation: $d_{p\omega\,min} \le d_p \le d_{p\,min}$ [9].

The obtained equations are used in this article to create the mathematical model of the hydro-cyclone separation of particles of TMF components by fractions with a given dispersion. The equation for the minimum diameter of a fully absorbed hydrophobic nanoparticle during hydro-cyclone coagulation (at $\omega_l = 0$ m) can be represented as

$$d_{p\omega min} = 24 \frac{\delta_{l\text{-}g}}{(\rho_p - \rho_g) V_l^2} \left(\cos\theta \sqrt{1 - K_\omega^2 \omega_l^4} - \sin\theta K_\omega^2 \omega_l^2 \right), \quad (1)$$

where ρ_p, ρ_g – particle and gas density, respectively, kg/m^3; V_l – liquid drop velocity, m/s; $\delta_{l\text{-}g}$ – surface tension coefficient at the interface between liquid-gas media, J/m^2; θ – contact angle at the interface between liquid-gas media, rad; K_ω – coefficient of influence of the angular velocity of rotation of a liquid drop on the minimum diameter of absorbed particles; ρ_l – density of a liquid drop, kg/m^3,

$$K_\omega = \frac{\pi \rho_l d_{p\,min}^3 \sin^4\theta}{8\delta_{l\text{-}g}\cos\theta}.$$

According to the results of experimental studies, it was found that the effective influence of swirl on coagulation is observed at a relative angular velocity of rotation of a liquid droplet $K_\omega \le \omega^2 \le 0.3$. The coefficient of variation of the median diameter of the absorbed nanoparticles of the TMF components from the angular velocity of rotation of the liquid droplets can be represented as follows

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$$K_{\omega}^{d} = 48\frac{\delta_{\text{l-g}}}{(\rho_{\text{p}} - \rho_{\text{g}})V_{\text{l}}^{2}} K_{\omega}^{2}\cos\theta\left(\omega_{\text{l}}^{3} - K_{\omega}^{2}\omega_{\text{l}}^{7}\right) - K_{\omega}\sin\theta\omega_{\text{l}}. \quad (2)$$

From equation (2) it follows that the angular velocity of rotation of the liquid droplets can be an effective control parameter in the process of hydro-cyclone separation of hydrophobic particles of TMF. A distinctive feature of hydro-cyclone separation is its high sensitivity to the dispersion of the median size of microparticles, since their separation is based on hydro-cyclone coagulation, while the size of the liquid droplets is significantly larger than the microparticles that they absorb.

We propose to use the Venturi hydro-cyclone separator for the recycling of TMF nanoparticles, which ensures their separation by fractions with a given median size and dispersion (Fig.1).

In terms of design the Venturi hydro-cyclone separator includes an air slide pipeline for the vertical movement of particles of bulk TMF materials and a device for the hydro-cyclone separation of particles by fractions due to inertial heterocoagulation by rotating droplets of liquid, consisting of a Venturi pipe, along the axis of which an aerator with vortex nozzles is installed in the throat section, whereas along the perimeter the ring around the receiving tank.

The Venturi hydro-cyclone separator contains a hopper feed unit 9 mounted above the separator collector 1. In the mixing chamber 11 there is a porous gas distribution partition 10 and a nozzle 12 for supplying compressed air and forming a fluidized bed of TMF material at the inlet of the collector 1. At the entrance to the Venturi pipe 7, a honeycomb 2 is installed to align the particle velocity. A rotary aerator with hydro-cyclone nozzles 5 is installed along the axis of the Venturi pipe, and a stratification collector 6 with a receiving tank 4 for collecting particles of TMF components along fractions with a drain valve 3 is located along its perimeter, and a hopper with a drain valve 8 for waste is installed at the separator output.

TMF from the hopper feed unit 9 are continuously sent to the mixing chamber 11, which is limited by the gas distribution partition 10. Compressed gas is supplied through the nozzle 12 under the TMF layer. The compressed gas is aerated to flow the bulk material to a fluidized state and fed through a collector 1, aligning the honeycomb 2 to the inlet of the Venturi pipe 7. Swirling liquid droplets from the nozzles of the aerator 5 wet the particles of the fluidized bed. Kinetic energy and the rotation speed of liquid droplets provide coagulation of TMF particles with a given minimum diameter.

The diameter of the aerator d_{a} and the Venturi pipe d_{V} in the throat section, as well as the energy of the liquid droplets, are chosen in such a way that coagulation of liquid droplets and particles of bulk TMF material with a given minimum median diameter occurs in the zone up to 80 % of the diameter of the Venturi pipe. Thus, in the annular zone of 20 % of the diameter of the Venturi pipe along its perimeter, the particles of the solid fuel component in the fluidized bed rise into the waste bin without being wetted due to insufficient energy of liquid droplets to overcome the aerodynamic energy barrier. It ensures the efficiency of particle separation by fractions with a given dispersion due to the exclusion of the unstable coagulation zone near the hopper from the separation process. The position of the separation collector 6 and the separation hopper 4 with respect to the plane of the nozzles of the hydro-cyclone aerator 5 is determined in accordance with the trajectories of the droplets of liquid with particles of bulk TMF mixed

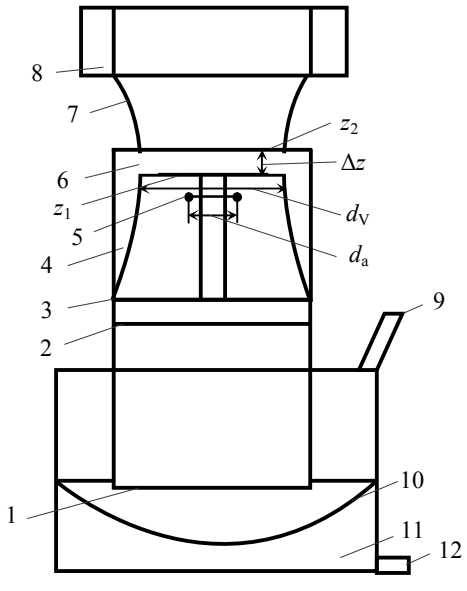


Fig.1. Layout of hydro-cyclone Venturi separator

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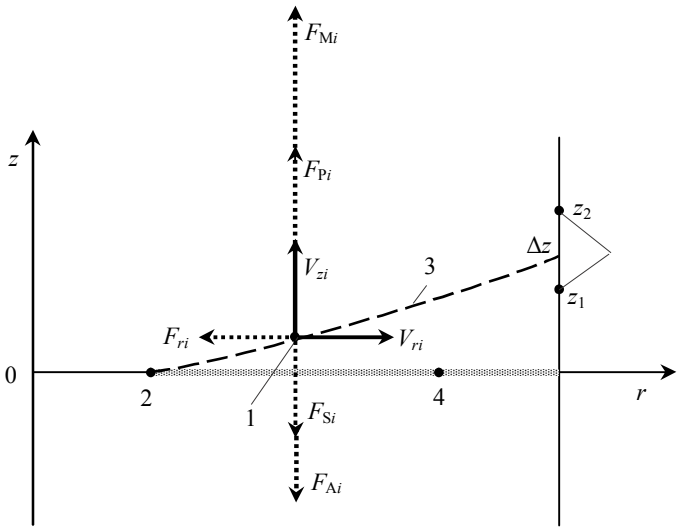


Fig.2. Motion scheme of a drop of liquid with integrated TMF component particle and forces acting on it under conditions of hydro-cyclone separation: $0z$ – axis of the Venturi separator; z_1, z_2 – coordinates of the lower and upper boundaries of the collector

1 – a drop of liquid with an integrated TMF component particle;
2 – the coordinate of the position of the hydro-cyclone aerator nozzle;
3 – trajectory of the motion of a liquid drop with a TMF component particle;
4 – fluidized bed plane

with them during inertial hydro-cyclone heterocoagulation calculated according to the proposed mathematical model. The moisture and final product are removed through the hopper with a drain valve 8.

We used the aerator rotating with angular velocity ω_a to increase the efficiency of separation by controlling the influence of the Magnus force on the speed of vertical movement of TMF particles absorbed by rotating drops of liquid in the Venturi separator.

The trajectory of the TMF nanoparticles is determined by the inertial interaction of nanoparticles, rotating liquid droplets, and fluidized-bed energy flow.

Research results. To construct a mathematical model of the unsteady hydrodynamic interaction of a liquid droplet with particles of components under conditions of high Reynolds numbers, we assume that during the motion of a liquid droplet it preserves a spherical shape d_l of the same density as the droplet fluid, in which the aerodynamic characteristics of motion in a gas medium correspond to the actual characteristics of the motion of the droplets at the same Reynolds numbers. The diameter d_l will be considered the aerodynamic diameter of the droplet [7].

To ensure the uniqueness and certainty of the solution, taking into account the proposed scientific hypothesis, we accept the condition under which the energy of the translational motion of liquid droplets is sufficient to overcome the aerodynamic energy barrier in the fluidized bed area of not more than 80 % of the diameter of the Venturi pipe, i.e. the diameter of the entrance to the receiving tank.

To design a system of equations for the motion of a liquid droplet with an integrated TMF component (Fig.2), we introduce the concepts of reduced aerodynamic diameter and density of nanoparticles and present them in the following form

$$d_\Sigma = \frac{6\sqrt[3]{\frac{\pi}{6}\left(d_l^3 + d_p^3\right)}}{\pi};\ \rho_\Sigma = \frac{d_l^3\rho_l + d_p^3\rho_p}{d_l^3 + d_p^3}, \tag{3}$$

where d_l – liquid droplet diameter, m; d_p – TMF microparticle diameter, m.

The equation of motion of the i-th particle when it is absorbed by a drop of liquid in the projection onto the $0r$ axis in the plane of the aerator's hydro-cyclone nozzles (in accordance with Newton's classical equation), taking into account Fig. 2, we can write in the form [4]:

$$m_i \frac{\partial V_{ri}}{\partial t} = F_{ri}, \tag{4}$$

where F_{ri} – drag force of a TMF component particle in a hydro-cyclone separator,

$$F_{ri} = k_i \frac{\pi}{4} d_{\Sigma i}^2 \rho_g V_{\Sigma i}^2;$$

k_i – drag coefficient for the i-th particle; $d_{\Sigma i}$ – diameter of the i-th particle, m; $V_{\Sigma i}$ – velocity of the i-th particle, m/s.

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The equation of motion of the i-th particle in the projection onto the 0z axis has the form

$$m_i \frac{\partial V_{zi}}{\partial t} = -F_{Ai} - F_{Si} + F_{Pi} - F_{Mi}, \quad (5)$$

where F_{Ai} – Archimedes force directed downward and acting on the i-th particle, which is an analog of gravity,

$$F_{Ai} = \frac{1}{6}\pi d_{\Sigma i}^3 \left(\rho_{\Sigma i} - \rho_g\right) g;$$

F_{Si} – Stokes resistance force due to air viscosity and physical properties of feed components,

$$F_{Si} = \frac{\varphi}{2}\mu_g \pi d_{\Sigma i} V_{zi};$$

F_{Pi} – pressure force of the compressed gas creating a fluidized bed,

$$F_{Pi} = C_c \rho_g V_{gi}^2 \frac{\pi}{4} d_{\Sigma i}^2;$$

F_{Mi} – Magnus force, the direction of which is determined by the angular speed of rotation of the aerator,

$$F_{Mi} = \frac{\pi}{8}\rho_g d_{\Sigma i}^3 \omega_{\Sigma i} V_{\varphi i};$$

C_c – pressure coefficient of compressed gas creating a fluidized bed acting on the i-th particle; V_g, V_{zi} – velocity of the compressed gas creating a fluidized bed, and the vertical component of the velocity of the i-th particle, m/s; g – gravity acceleration, m/s^2; φ_i – particle shape factor in Stokes law; μ – dynamic viscosity coefficient of air, kg/ms; $V_{\varphi i}$ – peripheral velocity of a liquid droplet with the i-th particle integrated into it in the plane of a fluidized bed.

Equation (4) is identical to the Boussinesq equation, which describes the hydrodynamically unsteady mode of inertial motion of a liquid droplet before and after coagulation of a nanoparticle of a TMF component [7]

$$\frac{\partial V_{ri}}{\partial t} = \frac{V_{ri}}{\tau_{pi}}, \quad (6)$$

where τ_{pi} – relaxation time of the i-th particle,

$$\tau_{pi} = \tau_{\Sigma i} = k_i \frac{2 + 3\overline{\mu}}{3 + 3\overline{\mu}} \frac{d_{\Sigma i}^2 \left(\rho_{\Sigma i} - \rho_g\right)}{18\mu_g};$$

$$\overline{\mu} = \frac{\mu_l}{\mu_g}.$$

The Boussinesq equation allows one to determine the relaxation time of a liquid drop and a nanoparticle and associates it with a drag coefficient. In this case, the drag coefficient in the Boussinesq equation corresponds to k_i in equation (4).

It has been experimentally established that the drag force increases substantially nonlinearly with an increase in the Reynolds number in the section of the over-Stokes motion, in contrast to its linear growth at numbers Re < 1 and with a simultaneous decrease in the relaxation time τ, which significantly complicates the determination of its actual value, thereby preventing the use of classical equations Stokes movement during coagulation [8].

Since in the equation of aerohydrodynamics of a liquid droplet in the horizontal plane of the Venturi separator (4), the drag coefficient of the gaseous medium to the motion of the liquid droplet k_i is a significant variable, we will establish its dependence on the physical values describing the hydrodynamic process of the inertial motion of the liquid droplet under the action of acceleration

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with the initial velocity V_0 in the form of a dimensionless simplex under steady-state conditions. We write the equation of the dependence of the coefficient k_i on independent variables in the form of a dimensionless power characteristic in the form

$$k_i = C\mu_g^{\alpha}\rho_g^{\beta}\sqrt{V_{ri}^2 + 0{,}25d_{\Sigma i}^2 \sin\theta^2\omega_1^2}^{\gamma} d_{\Sigma i}^{\varepsilon}\left(\frac{\partial V_{ri}}{\partial t}\frac{\rho_1-\rho_g}{\rho_g}\right)^{\varphi} V_0^{\psi}. \quad (7)$$

To determine the similarity indicators, we use a matrix of independent dimensions, the rank of which is three [1]. Given a homogeneous system of linear equations composed of exponents of equation (7), the matrix of similarity indicators will take the form

	μ_g	ρ_g	$\sqrt{V_1^2 + 0{,}25\omega_1^2 d_{\Sigma i}^2 \sin\theta\omega}$	d_1	$\frac{\partial V_{ri}}{\partial t}\frac{\rho_1-\rho_g}{\rho_g}$	V_0
π_1	-2	2	2	2	0	0
π_2	0	0	0	1	1	-2
π_3	-2	2	0	2	0	$+2$

(8)

Finding the determinants of the matrix (8), we obtain three indicators of the similarity of the physical process of steady state aerohydrodynamic motion of a liquid droplet in a gaseous medium at large Reynolds numbers

$$\pi_1 = \frac{\rho_g^2\left(V_{ri}^2 + 0{,}25\omega_1^2 d_{\Sigma i}^2 \sin\theta\omega\right)d_{\Sigma i}^2}{\mu_g^2} = \mathrm{Re}_{\Sigma\omega i}^2; \quad (9)$$

$$\pi_2 = \frac{P_i}{\rho_g V_0^2} = \frac{\frac{\partial V_{ri}}{\partial t} d_{\Sigma i}\left(\rho_1-\rho_g\right)}{\rho_g V_0^2} = \mathrm{Eu}_i; \quad (10)$$

$$\pi_3 = \frac{\rho_g^2 V_0^2 d_{\Sigma i}^2}{\mu_g^2} = \mathrm{Re}_{\Sigma 0 i}^2. \quad (11)$$

Thus, equation (7) in the criteria form takes the form

$$k_i = C\left(\mathrm{Re}_{\Sigma\omega i}^2\right)^a \mathrm{Eu}_i{}^b \left(\mathrm{Re}_{\Sigma 0 i}^2\right)^c. \quad (12)$$

The numerical values of the proportionality coefficient C and the exponents a, b, c in equation (12) are determined from the equation of steady motion of a liquid drop in a gas medium

$$\frac{m_i \partial V_{ri}}{\partial t} = \mathrm{Eu}_i \rho_g V_{0i}^2 \frac{\pi d_{\Sigma i}^2}{4}. \quad (13)$$

Taking into account equations (4, 13), the expression for the aerodynamic drag coefficient is obtained in the form

$$k_i = \frac{2}{3}\frac{\mathrm{Eu}_i}{\mathrm{Re}_{\Sigma\omega i}^2 \mathrm{Re}_{\Sigma 0 i}^2}. \quad (14)$$

After the appropriate conversions of equations (12-14) concerning the Reynolds number, we obtain the equation for determining the relaxation time of the dispersed system of «a drop of liquid – a TMF microparticle» in the form

$$\tau_{pi} = \frac{2}{9}\frac{d_{\Sigma i}^2\left(\rho_{\Sigma i}-\rho_g\right)\mathrm{Re}_{0i}^2}{\mathrm{Eu}_i \mu_g \mathrm{Re}_{\Sigma i}^2}. \quad (15)$$

Considering that the Reynolds and Euler criteria are functionally interconnected and continuously and substantially change along the inertial path of a liquid drop under the conditions of the over-Stokes motion, we consider the possibility of solving the problem by averaging kinematic pa-

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rameters. The classical theory of hydrodynamic motion under steady-state conditions makes it possible to obtain in quadrature expressions for the relaxation time of liquid droplets and TMF particles depending on the kinematic parameters of the flow. In [8], we obtained the expression for changing the relaxation time for large Reynolds numbers by averaging their values. Taking into account the indicated data, the expression for the Euler criterion under conditions of unsteady hydrodynamic motion at the stage of relaxation of the dispersed «a drop of liquid – a TMF microparticle» system can be represented as

$$\mathrm{Eu}_i = \frac{18\,\mathrm{Re}_{\Sigma i}}{\mathrm{Re}_{0i}^2}\left(1+0.15\,\mathrm{Re}_{0i}^{0.687}\right). \tag{16}$$

Thus, taking into account the data presented in [7, 8], the average value of the Euler criterion in the relaxation region of the dispersed «a drop of liquid – a TMF microparticle» system can be represented as

$$\mathrm{Eu}_{\mathrm{av}\,i} = \frac{6}{\mathrm{Re}_{0i}}\left(1+0.07\,\mathrm{Re}_{0i}^{0.687}\right). \tag{17}$$

Given the relationship between the Reynolds criterion and the drag coefficient to the flow of the dispersed «a drop of liquid – a TMF microparticle» system in a gaseous medium, we write the expression for the average drag coefficient in equation (7) as

$$k_{avi} = \frac{72}{\mathrm{Re}_{0i}}\left(1+0.07\,\mathrm{Re}_{0i}^{0.687}\right). \tag{18}$$

After the appropriate conversions, we obtain the expression for the average relaxation time of the dispersed «a drop of liquid – a TMF microparticle» system

$$\tau_{\Sigma \mathrm{av}\,i} = 4d_{\Sigma i}^2\frac{\left(\rho_{\Sigma i}-\rho_{\mathrm{g}}\right)\left(1+0{,}07\,\mathrm{Re}_{0i}^{0.687}\right)\left(3+3\overline{\mu}\right)}{\mathrm{Re}_{0i}\left(2+3\overline{\mu}\right)\mu_{\mathrm{g}}} = K_{\tau}d_{\Sigma i}^2. \tag{19}$$

We can see from the equation that the average relaxation time of liquid droplets with integrated TMF components is a function of the square of the diameter of the dispersed «a drop of liquid – a TMF microparticle» system, this fact can be used to develop a technology for the efficient classification of finely dispersed bulk TMF.

Considering the homogeneity of the equations of translational and rotational motion of bodies in a wide range of Reynolds numbers, the obtained ratio for the average values of the drag coefficient and relaxation time can be applied to the angular velocity of rotation of liquid droplets with integrated TMF particles caused by the rotation of the aerator. In this case, the speed of vertical movement of the dispersed «a drop of liquid – a TMF microparticle» system in a fluidized bed can be represented as

$$V_{zi} = \frac{\frac{1}{2}C_{\mathrm{c}}\rho_{\mathrm{g}}V_{\mathrm{g}i}^2 d_{\Sigma i} - \frac{1}{3}d_{\Sigma i}^2\left(\rho_{\Sigma i}-\rho_{\mathrm{g}}\right)g - 4\cdot 10^{-4}\omega_0\omega_{\mathrm{a}}\rho_{\mathrm{g}}\rho_{\overline{\Sigma}i}d_{\mathrm{p}i}^5 d_{\Sigma i}^{-2}\left(1+\frac{d_{\mathrm{a}}}{d_{\mathrm{V}}}\right)}{\varphi\mu_{\mathrm{g}}}, \tag{20}$$

where $\rho_{\overline{\Sigma}i} = \frac{\rho_{\Sigma i}}{\rho_{\mathrm{l}}}$.

From the analysis of equation (20), we can see that by choosing the direction of rotation of the aerator, it is possible to achieve a significant deceleration of vertical movement of TMF particles, which varies in proportion to the square of their diameter, which improves separation efficiency.

Thus, the proposed version of the stepwise averaging of the values of the drag coefficient to the motion of the dispersed «a drop of liquid – a TMF microparticle» system in a gaseous medium and its relaxation time makes it possible to use with a sufficient accuracy rate the equations of classical aero hydrodynamics of the steady motion of a liquid droplet in the range of

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Reynolds numbers up to 10^4 over their inertial free pathlength to develop a mathematical model of inertial hydro-cyclone separation.

Generally, the technical design assignment for the Venturi hydro-cyclone separator has the value for the recycling efficiency for has the mass of bulk materials TMF Q, t/h.

The diameter of the Venturi separator d_V at a given performance value is determined by the formula

$$d_V = 2\sqrt{\frac{K_\rho Q}{\pi \rho_p V_z}}, \quad (21)$$

where K_ρ – density coefficient of the fluidized bed of TMF particles, $K_\rho = \rho_p / \rho_{fb}$; ρ_{fb} – density of particles of TMF in a fluidized bed state.

The velocity of liquid droplets at the outlet of aerator hydro-cyclone nozzles, characterizing the energy parameters, is determined by the formula

$$V_0 = 3.2\sqrt{\frac{3}{2}\frac{\delta_{l-g}\cos\theta}{\rho_\Sigma d_{\Sigma\min}} + \frac{d_V - d_a}{\tau_{\Sigma av}}}. \quad (22)$$

The angular velocity of rotation of the liquid droplets and the coefficient of variation of the median diameter of the absorbed TMF microparticles, which determine the target fraction during separation, are found by the formulas (1, 2).

The coordinates of the input collector of the separator z_1, z_2 of the receiving tank for a given median diameter d_m and its dispersion σ_m are determined by the formulas:

$$z_1 = \frac{C_c \rho_g V_g^2 d_{\Sigma\max} - \frac{2}{3}(\rho_{\Sigma\max} - \rho_g) g d_{\Sigma\max}^2 - 5\cdot 10^{-4}\omega_0\omega_a\rho_l\rho_{\Sigma\max}^{-1} d_{4\max}^5 d_{\Sigma\max}^{-3}\left(1+\frac{d_a}{d_k}\right)}{10\varphi\mu_g \ln\frac{V_0}{\sqrt{6\delta_{l-g}\cos\theta\rho_{\Sigma\max}^{-1} d_{\Sigma\max}^{-1}}}}\tau_{\Sigma av\,\max}; \quad (23)$$

$$z_2 = \frac{\frac{1}{2}C_c\rho_g V g_g^2 d_{\Sigma\min} - \frac{1}{3}(\rho_{\Sigma\min} - \rho_g) g d_{\Sigma\min}^2 - 5\cdot 10^{-4}\omega_0\omega_a\rho_l\rho_{\Sigma\min}^{-1} d_{4\min}^5 d_{\Sigma\min}^{-3}\left(1+\frac{d_a}{d_k}\right)}{\varphi\mu_g \ln\frac{V_0}{\sqrt{6\delta_{l-g}\cos\theta\rho_{\Sigma\min}^{-1} d_{\Sigma\min}^{-1}}}}\tau_{\Sigma av\,\min}, \quad (24)$$

where $d_{\Sigma\min} = d_{\Sigma m} - 3\Sigma_m$; $d_{\Sigma\max} = d_{\Sigma m} + 3\Sigma_m$.

Figures 3-6 show the results of experimental studies and their comparison with calculations in the proposed mathematical model.

The data of experimental studies (Fig.3) compared to the calculations in the proposed mathematical model confirm the efficiency of controlling the minimum diameter of absorbed hydrophobic TMF particles using the angular velocity of rotation of liquid droplets during hydro-cyclone separation.

The results of experiments and calculations (Fig.4, 5) open manifold possibilities for controlling the geometric parameters and the location of the input collector of the receiving tank using the angular velocity of rotation of the liquid droplets to ensure the required efficiency of separation of TMF particles by fractions.

A significant change in the relaxation time, according to the experimental results and the calculated data shown in Fig.6, allows us to achieve high separation efficiency for a given dispersion of the median sizes of TMF micro particles due to the significant effect of a change in their nominal diameter on the inertial forces that determine the trajectory of the dispersed «a drop of liquid – a TMF microparticle» system.

The results of experimental studies of the Venturi hydro-cyclone separator carried out at the «SMK-TEST» company confirmed the convergence with the proposed mathematical model, which is sufficient for engineering analysis.

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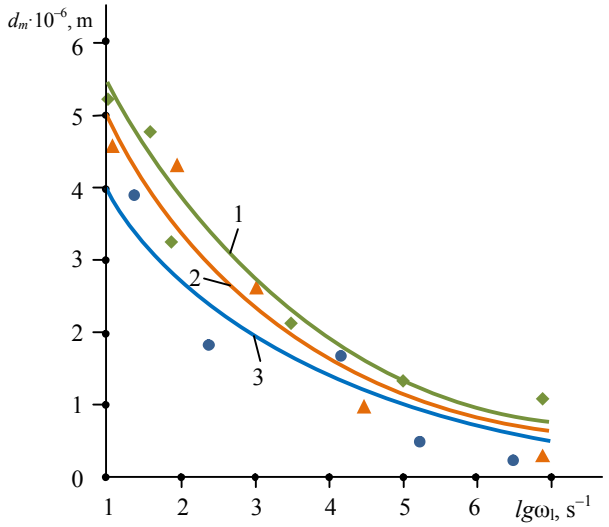


Fig.3. Dependence of the median diameter of the wetting of TMF particles on the angular velocity of rotation of liquid droplets during hydro-cyclone separation

1 – coal; 2 – silicon oxide; 3 – aluminum oxide

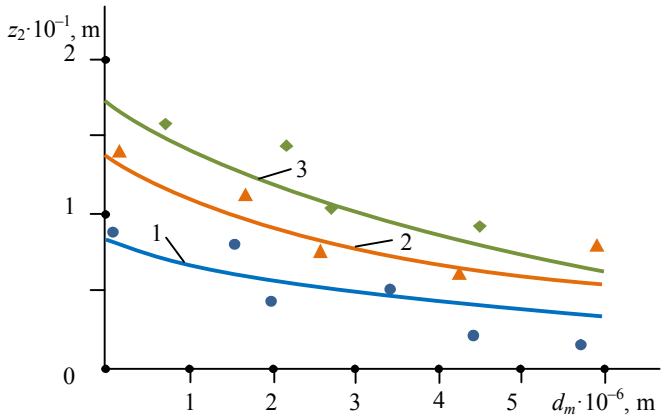


Fig.4. Dependence of the coordinate of the input collector from the separated median particle diameter.

1 – $\omega_a = 50\ s^{-1}$; 2 – $\omega_a = 100\ s^{-1}$; 3 – $\omega_a = 150\ s^{-1}$

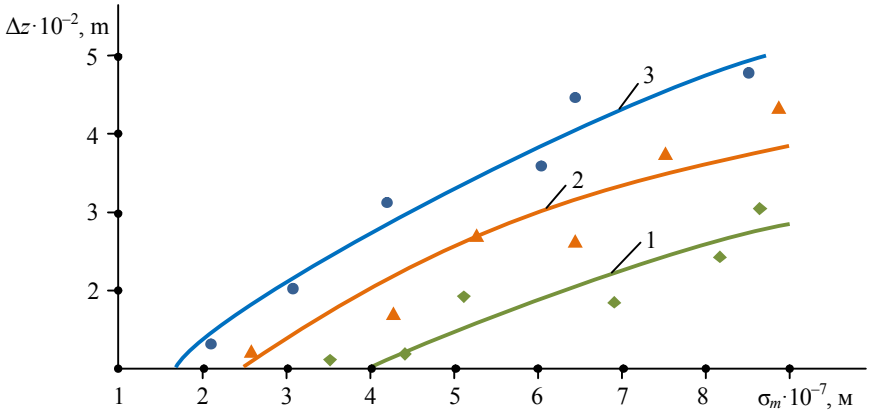


Fig.5. Dependence of the width of the input collector on the variance of the separated median particle size

1 – $\omega_a = 50\ s^{-1}$; 2 – $\omega_a = 100\ s^{-1}$; 3 – $\omega_a = 150\ s^{-1}$

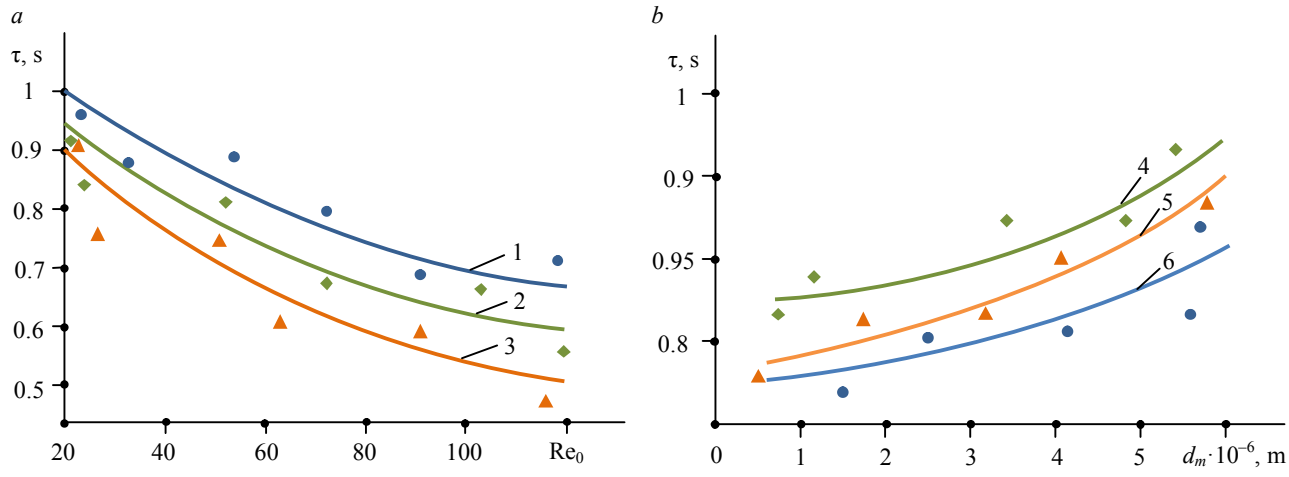


Fig.6. Dependence of the average relaxation time of a liquid droplet with an integrated one: *a* – a particle of aluminum oxide on the Reynolds criterion and *b* – a particle of a TMF component on the median diameter

1 – $d_\Sigma 3.5 \cdot 10^{-4}$, m; 2 – $d_\Sigma 2.5 \cdot 10^{-4}$, m; 3 – $d_\Sigma \cdot 10^{-4}$, m; 4 – coal; 5 – silicon oxide; 6 – aluminum oxide; $Re_0 = 40$

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Conclusion. The results of the research allow us to draw the following conclusions applicable in scientific and practical areas:

1. The diameter of completely absorbed solid particles depends on the angular velocity of rotation of the liquid droplets during the hydro-cyclone heterocoagulation, which allows using this physical process for the classification of micro and nanoparticles.

2. The relaxation time of liquid droplets with TMF micro and nanoparticles integrated during the hydro-cyclone separation, and hence the inertial forces of non-stable interaction with rotating liquid droplets, depend on the median diameter, which is the key factor for achieving high separation efficiency.

3. The geometric parameters of the Venturi hydro-cyclone separator are determined by the required productivity and energy properties of the aerator.

4. The hydro-cyclone separation allows us to classify TMF microparticles in the range of $0.5\text{-}5 \cdot 10^{-6}$ m with a dispersion of median sizes of not more than 20 %.

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