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FEATURES OF MATHEMATICAL MODELING OF NATURAL GAS PRODUCTION AND TRANSPORT SYSTEMS IN THE RUSSIA'S ARCTIC ZONE

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The necessity of accounting for real gas properties, thermal interaction with permafrost rocks and the possibility of formation (dissociation) of gas hydrates in these objects for adequate description of the operation of gas wells and main gas pipelines in the regions of the Far North by appropriate mathematical models is shown. Mathematical models that take into account the non-isothermal gas flow within the framework of pipe hydraulics, the change of the area of tube cross-section due to the formation of hydrates and the dependence of the heat transfer coefficient between gas and hydrate layer on the varying flow area over time are proposed. The corresponding conjugate problem of heat exchange between the imperfect gas in the well and the environment (rocks) is reduced to solving differential equations describing the non-isothermal flow of gas in the pipes and the heat transfer equations in rocks with the corresponding conjugation conditions. In the quasi-stationary mathematical model of hydrate formation (dissociation), the dependence of the gas-hydrate transition temperature on the pressure of gas is taken into account. Established that the formation of hydrates in wells, even at low reservoir temperatures and a thick layer of permafrost, takes a fairly long period of time, which allows to quickly prevent the creation of emergency situations in gas supply systems. Some decisions taken in the design of the first section of the main gas pipeline «Power of Siberia» have been analyzed by methods of mathematical modeling. In particular, it is shown that if the gas is not dried sufficiently, the outlet pressure may drop below the allowable limit in about 6-7 hours. At the same time, for completely dry gas, it is possible to reduce the cost of thermal insulation of the pipeline at least two fold.

Key words: natural gases hydrates, permafrost rocks, conjugate heat exchange problem, gas well, gas pipeline, heat insulation, computer experiment

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Introduction. The technological regimes of gas production in the Northern regions are largely determined by such natural factors as low temperatures and the presence of a thick layer of permafrost. Their consequence is the complications caused by the possibility of hydrate formation both in the bottom hole area and in the wellbore. The first leads to a decrease in the productivity of the wells, the second – to the complete gas flow failure. Such contingency situations could have the most severe consequences. Currently, the only way to control this undesirable phenomenon is to inject methanol or other hydrate inhibitors into the wells, which is ineffective, since methanol is removed from wells along with the produced gas, which significantly increases the cost of production and transport of gas. Consequently, the actual problem is the choice of such gas production flow rate in which these contingency situations can be eliminated or their effect on the reliability of gas supply reduced.

Due to thermal interaction with rocks, throttling and adiabatic expansion (compression), the temperature of the produced gas changes, and the temperature of the rocks changes accordingly. These processes are interrelated, and therefore this problem can be solved only in the conjugate formulation – while simultaneously determining the change in the gas temperature in the well and the temperature field of the surrounding rocks. Thus, the mathematical model of the process should include: the heat conduction equation describing the heat propagation in the rocks, taking into account their possible thawing-freezing, the equation for the non-isothermal flow of gas in the well and the necessary boundary and initial conditions determined by the nature of the conjugation of heat flows on the wellbore wall.

The problem of hydrate formation in a gas well and the algorithm for computational solution. The necessary analysis was carried out in the framework of the mathematical model of hydrate formation during gas flow in pipes, proposed in the monograph [12] and modified in subsequent publications of the authors [1, 3, 4, 8, 14]. In this model, the flow of gas in the tube of a variable cross-section with time is described in the quasi-stationary approximation, since the rate of the transient process in the gas flow is much higher than the rate of change in the temperature of the surrounding frozen rocks due to thermal conductivity. This means that time is included into the equations of pipe hydraulics parametrically through the variable ambient temperature and through changes in the area of the flow section. The additional assumption used in the model is associated with a low gas flow rate in comparison with the speed of sound [12]. The heat propagation in permafrost rocks is described in the framework of the Stefan problem. These equations are supplemented by the conditions of conjugation of heat fluxes. In the quasi-stationary mathematical model of formation (dissociation) of hydrates in the well, the dependence of the heat transfer coefficient between the gas flow and the inner wall of the pipe on the flow area changing with time is taken into account, and also the dependence of the temperature of the gas-hydrate phase transition on the gas pressure.

To describe the formation and deposition of hydrates, a quasi-stationary mathematical model is used [1, 3, 4, 8, 12, 14] in which the motion of imperfect gas in pipes is described within the framework of pipe hydraulics, and the dynamics of hydrate formation – within the framework of the generalized Stefan problem in which the temperature of the gas-hydrate phase transition essentially depends on the pressure in the gas flow. In this model, based on the laws of conservation of mass and energy for the gas flow, the equations of continuity, motion and energy of a gas are reduced to a system of two ordinary nonlinear differential equations for pressure and temperature:

$$\frac{dp}{dx} = -\rho_g g \sin\varphi - \frac{\sqrt{\pi}\psi M^2}{4\rho_g S^{2,5} S_0^{2,5}}; \qquad (1)$$

$$\frac{dT}{dx} - \varepsilon\frac{dp}{dx} = \frac{\pi D \alpha}{c_p M}\left(T_e - T\right) - \frac{g}{c_p}\sin\varphi, \qquad (2)$$

where p – pressure; x – coordinate along the axis of the pipe; ρ_g – gas density; g – gravitational acceleration; φ – angle of slope of the pipe, measured from a fixed horizontal plane; ψ – coefficient of hydraulic resistance; $M = \rho_g \upsilon S S_0$ – constant mass flow rate of gas; S – a dimensionless cross-section; S_0 – dimensional cross-section before formation of hydrates; T – gas temperature ; D – diameter of the flow section; α – total heat transfer coefficient; c_p – specific heat of gas at constant pressure; T_e – temperature of surrounding rocks; υ – speed of gas flow.

The gas density and throttling coefficient ε are related to pressure and temperature by equations

$$\rho_g = \frac{p}{ZRT}; \ \varepsilon = \frac{RT^2}{c_p p}\left(\frac{\partial Z}{\partial T}\right)_p, \qquad (3)$$

where $Z = Z(p, T)$ – gas imperfection coefficient, an empirical function, depending on the ratio of pressure and temperature to their critical values; $R = 8.314/\mu$ – gas constant; $\mu = \sum_{i=1}^{n} y_i \mu_i$ – molar mass of gas mixture; y_i and μ_i – volume fraction and molecular weight of the i-th component of natural gas.

It was shown in [2] that at high pressures and temperatures the Latonov-Gurevich equation [7] is in good agreement with the experimental data

$$Z = \left(0.17376 \ln\frac{T}{T_c} + 0.73\right)^{\frac{p}{p_c}} + 0.1\frac{p}{p_c},$$

where critical pressure and temperature of gas mixture are determined by the Kay rule [15]: $p_c = \sum_{i=1}^{n} y_i p_{ci}$, $T_c = \sum_{i=1}^{n} y_i T_{ci}$ in which p_{ci} and T_{ci} are critical parameters of the i-th component of natural gas.

The equation describing the change in the area of the flow section of the well S over time is recorded in the non-dimensional form:

$$\frac{dS}{d\tau} = b_2 \frac{T_e - T_h(p)}{1 - b_2 \ln S} - b_1 \sqrt{S}\left(T_h(p) - T\right), \tag{4}$$

where $\tau = \lambda_h T_c / \rho_h q_h D_0^2 t$ – non-dimensional time; $b_2 = \alpha_2 D_0 / 4\lambda_h$; $T_h(p) = a \ln p + b$ – equilibrium temperature of hydrate formation; $b_1 = \alpha_1 D_0 / 4\lambda_h$; t – time; λ_h, ρ_h – coefficient of thermal conductivity and density of hydrate; q_h – specific heat of hydrate formation; D_0 – diameter of the pipe before formation of hydrate; α_2 – coefficient of heat exchange between hydrate layer and rocks; α_1 – coefficient of heat exchange between gas and hydrate layer.

The empirical coefficients a and b are found by approximating the thermodynamic equilibrium curve of hydrate formation, determined by the Sloane method [16] from the known gas composition. In equation (4) all temperature values are assigned to the critical gas temperature T_c.

The initial conditions for equations (1), (2), and (4) can be formulated as

$$p(0) = p_0,\ T(0) = T_0,\ S(0) = \text{const}. \tag{5}$$

In equation (4) the coefficient α_1 depends on the time-varying flow area of the pipe S. To derive the corresponding relation a known semi-empirical formula for heat transfer coefficient is used for turbulent gas flow in pipes [11]

$$\text{Nu} = 0.023\text{Pr}^{0.43}\text{Re}^{0.8}, \tag{6}$$

where $\text{Nu} = \alpha_1 D / \lambda_g$ – the Nusselt number; $\text{Pr} = \eta_g c_p / \lambda_g$ – the Prandtl number; $\text{Re} = \upsilon D \rho_g / \eta_g$ – the Reynolds number; η_g and λ_g – dynamic viscosity and thermal conductivity of gas.

Using the expression for the mass flow rate of gas and formula (6) we obtain the required relation in the form

$$\frac{\alpha_1 D_0}{\lambda_g} = 0.023\text{Pr}^{0,43}\left(\frac{M}{D_0 \eta_g}\right)^{0.8}\left(\frac{4}{\pi}\right)^{0.8}\frac{1}{S^{0.9}}. \tag{7}$$

In those sections of the well where the hydrate layer is formed, i.e. where the dimensionless value of the cross-section $S < 1$, the heat transfer coefficient in equation (2) is calculated by formula (7). In this case, the temperature of the T_e rocks is replaced by the equilibrium temperature of hydratey formation T_h.

Equations (2) and (4) contain the temperature of the rocks T_e, determined from the solution of the differential equation of heat conduction, which is written in a form convenient for numerical solution by the front-capturing method with temperature smoothing of the discontinuous coefficients in the neighborhood of the «ice-water» phase transition:

$$\tilde{C}(T_e)\frac{\partial T_e}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda(T_e)\frac{\partial T_e}{\partial r}\right) \text{ when } r_1 < r < r_2,\ t > 0, \tag{8}$$

where $\tilde{C}$ – volumetric heat capacity of rocks; r – radial coordinate; λ – coefficient of thermal conductivity; r_1 – outer radius of the well; r_2 – radius of thermal influence.

The equation (8) is written in the assumption that heat flux in each cross section of the tube propagates strictly radially. The connection between sections is realized through the solutions of equation (2) and the boundary condition on the outer wall of the well

$$\lambda(T_e)\frac{\partial T_e}{\partial r} = \alpha(T_e - T) \text{ when } r = r_1. \tag{9}$$

In the distance from the well bottom hole to the lower boundary of permafrost, the coefficients in equation (8) are constant and its solution can be performed by standard methods. In the permafrost region this problem is complicated, since it is necessary to take into account the «ice-water» phase transition. For a numerical solution of the Stefan-type problem methods based on the approach outlined in the monograph [13] are used. For these methods the authors of [5, 10] developed an economical difference scheme which makes it possible to use homogeneous difference schemes. In this case, the latent heat of the phase transition $W = q_{ph}\rho_r\omega_r$ is introduced as the concentrated heat capacity in the coefficient $\widetilde{C}(T_e)$. Here q_{ph} – specific heat of the «ice-water» phase transition; ρ_r and ω_r – density and weight moisture of rocks.

On the conventional radius of thermal influence we accept the condition of thermal insulation

$$\frac{\partial T_e}{\partial r} = 0 \text{ when } r = r_2. \tag{10}$$

The initial distribution of the temperature of the rocks at the time of starting well after a long period of inactivity is given in the form

$$T_e = \begin{cases} T_{e0} - \Gamma x, & 0 < x < L - H; \\ T_{fr}, & L - H < x < L, \end{cases} \tag{11}$$

where T_{e0} – temperature at the well bottom hole; Γ – geothermal gradient; L – well depth; H – depth of permafrost; T_{fr} – frozen rock temperature.

Therefore, to determine the change of gas temperature and area of the flow cross section of well during its thermal interaction with the rocks, it is necessary to solve equations (1)-(11) simultaneously.

The algorithm of numerical solution of the conjugated problem of heat exchange of a well with rocks is described as follows:

I. Specify geometric and physical parameters as well as the initial conditions (5) and (11).

II. For a fixed cross-sectional area calculate gas pressure $p(x)$ and temperature $T(x)$ in the borehole solving equations (1)-(3) by the fourth-order Runge – Kutta method.

III. Making a time step in equations (4) and (7), find a new value of the flow cross section. Here the coordinate x enters the equation (4) as a parameter.

IV. Find the temperature distribution in the rocks solving the problem (8)-(11). Since the smoothed coefficients in Eq. (8) depend on the temperature, the resulting difference problem will be nonlinear and its solution is found by a simple iteration method using sweeping algorithms.

At each time step, paragraphs II-IV are repeated.

In constructing the computational algorithm, a significant difference in time scales for processes occurring in the well and in the rocks was taken into account: the transient processes in the well terminate in a short time and therefore gas temperature tracks the slow changes of the rock temperature.

The following values of the parameters corresponding to two deposits of the Republic of Sakha (Yakutia): were used in the calculations:

1) Sredne-Vilyuisky gas field $\alpha = 5.82$ W/(m^2·K); $D_0 = 0.1$ m; $\varphi = 90^\circ$; $\psi = 0.02$; $\rho_h = 920$ kg/m^3; $q_h = 510000$ J/kg; $\lambda_h = 1.88$ W/(m·K); $\lambda_g = 0.0307$ W/(m·K); $c_p = 2300$ J/(kg·K); $\eta_g = 1.3 \cdot 10^{-5}$ Pa·s; $R = 449.4$ J/(kg·K); $p_0 = 240 \cdot 10^5$ Pa; $T_0 = 323$ K; $p_c = 46.573 \cdot 10^5$ Pa; $T_c = 205.239$ K; $a = 7.009$ K; $b = 178.28$ K; $L = 2550$ m; $H = 500$ m; $T_{e0} = 328$ K; $\Gamma = 0.0277$ K/m; $T_{fr} = 271.15$ K; $T_{ph} = 273.15$ K; $q_{ph} = 334400$ J/kg; gas composition (volume fractions), %: CH_4 90.34, C_2H_6 4.98, C_3H_8 1.74, iC_4H_{10} 0.22, nC_4H_{10} 0.41, C_5H_{12+} 1.55, CO_2 0.28, N_2 0.48;

2) Otradninsky gas field $R = 438.3$ J/(kg·K); $D_0 = 0.146$ m; $p_0 = 188.35 \cdot 10^5$ Pa; $T_0 = 286.35$ K; $p_c = 44.71 \cdot 10^5$ Pa; $T_c = 195.376$ K; $a = 6.635$ K; $b = 182.951$ K; $L = 2480$ m; $H = 680$ m; $T_{e0} = 286.48$ K; $\Gamma = 0.0085$ K/m; gas composition (volume fractions), %: CH_4 83.15, C_2H_6 4.16, C_3H_8 1.48, iC_4H_{10} 0.17, nC_4H_{10} 0.50, iC_5H_{12} 0.12, nC_5H_{12} 0.17, C_6H_{14} 0.17, C_7H_{16+} 0.28, CO_2 0.07, N_2 9.50, H_2 0.02, He 0.21; the other parameters have the same values as in the first variant.

It can be seen that at approximately equal depth of the productive horizon, the composition of natural gas, as well as the reservoir and geothermal conditions of thesefields are significantly different. The characteristics of rocks are assumed to be the same, the index *th* corresponds to thawed zone, the index *f* to frozen area of rocks (see table).

At the initial stage the optimal mass flow rate gas corresponding to the minimum of heat losses in the absence of a hydrate layer was calculated. For the Sredne-Vilyuisky field it was equal to 9 kg/s, and for Otradninsky field it corresponds approximately to the maximum free flow rate of the well, and therefore calculations were carried out at a mass flow rate of 2.86 kg/s, which corresponds to an operating production rate of 187,000 m^3/day. In the subsequent computational experiment, the initial values of the free section of the well and the mass flow varied.

Physical characteristics of rocks

Depth interval, m	ρ_r, kg/m^3	ω_r, units	λ_{th}, W/(m·K)	λ_f, W/(m·K)	C_{th}, kJ/(m^3·K)	C_f, kJ/(m^3·K)
0-86	2000	0,120	1,69	1,93	2570	2310
86-*H*	2000	0,120	1,62	1,86	2680	2420
H-980	2300	0,060	2,00	–	2440	–
980-1831	2350	0,055	2,27	–	2420	–
1831-2561	2380	0,053	2,38	–	2420	–
2561-*L*	2330	0,057	2,10	–	2440	–

The most interesting results were obtained for the Sredne-Vilyuisky field. They are presented in Fig.1-5. First of all we note that at an optimal flow rate a hydrate plug is formed near the wellhead, and its lower boundary is much higher than the base of permafrost, as it is clearly seen in Fig.1, *a*, where the point of intersection of the gas temperature (curves 2 and 3) and the equilibrium hydrate formation temperature (curve 4) corresponds to a depth of 2550 – 2505 = 45 m, in this interval the gas temperature becomes lower than the temperature of hydrate formation. Reduction of the flow cross section is accompanied by a sharp drop in pressure near the wellhead (Fig.1, *b*). For the conjugated problem gas temperature and pressure in the upper part of the well bore are slightly higher, and the interval of hydrate plug formation is slightly less (33 m) than at a constant rock temperature (curves 3 in Fig.1, *a* and 1, *b*).

Consider changes of the flow cross section for two values of mass flow rate, when initially the well is free of hydrates, i.e. $S(0) = 1$ (Fig.2). It can be seen that for the conjugate statement the time for the formation of hydrate plugs increases substantially (see surface 2) in contrast to the case when the temperature of the surrounding rocks is assumed to be unchanged (see surface 1). With the increase of mass flow rate the duration of the process of full hydrate blockage of the well s increases: for an optimal flow rate it is approximately 423 hours for the conjugate and 251 hours – for the non-conjugate statement, and for a lower flow rate, respectively, 13 and 9 hours. The most intensive thawing occurs near the bottom of the permafrost, which is due to the relatively high temperature of gas, and the thawing radius is approximately proportional to the mass flow rate: 1.3 m at 9 kg/s, 0.25 m at 2 kg/s (compare Fig.3, *a* and Fig.3, *b*).

The situation becomes less predictable if at the initial moment the well is only half hydrate-free – $S(0) = 0.5$ (Fig.4, 5). In this case, the optimal gas flow rate is 4.8 kg/s. With this flow rate the hydrate plug near the wellhead is formed after 173 h (Fig.4, *a*, surface 2) for the conjugate problem and after 109 h (Fig.4, *a*, surface 1) for the non-conjugate problem. The lower boundary of the plug

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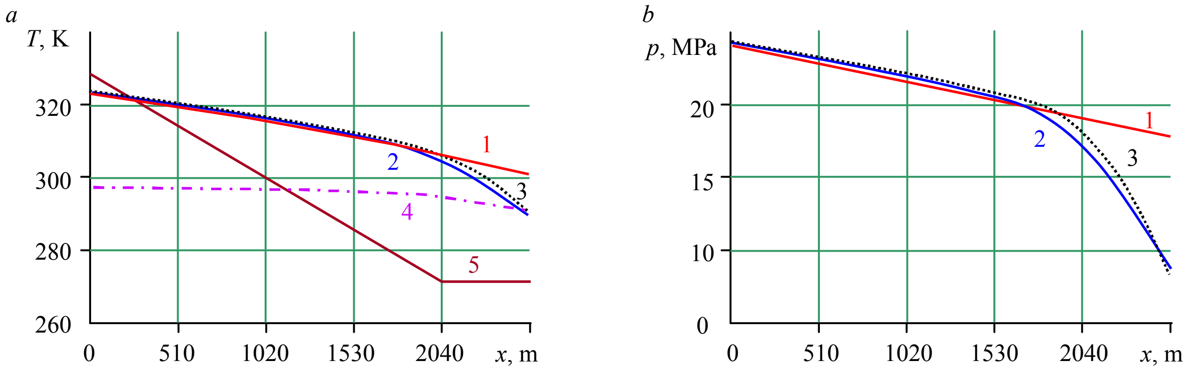

Fig.1. Change in temperature (*a*) and pressure (*b*) of gas with a depth of well for the Sredne-Vilyuisky field at $M = 9$ kg/s

1 – $t = 0.34$ h; 2 – $t = 251.3$ h (non-conjugatestatement); 3 – $t = 422.9$ h (conjugatestatement); 4 – equilibrium temperature of hydrate formation; 5 – initial temperature of rocks

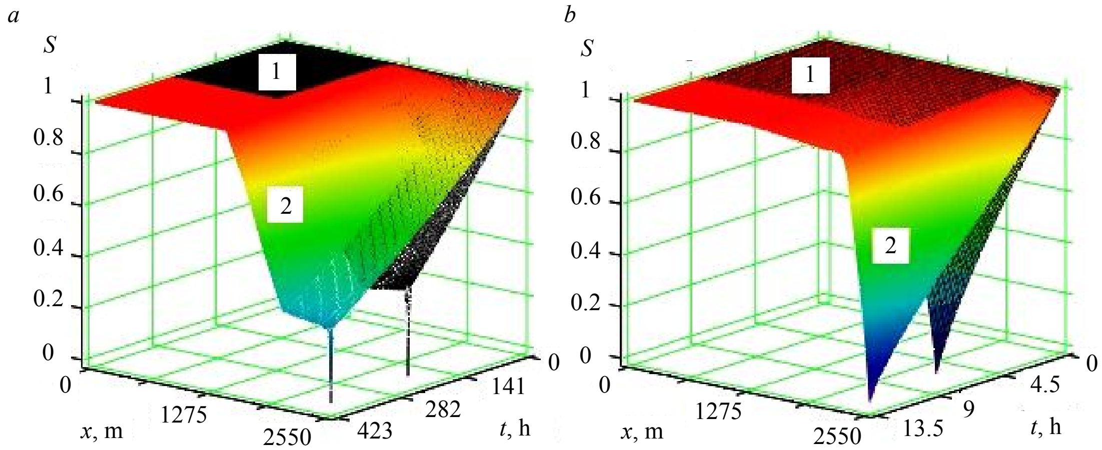

Fig.2. Change of the flow area of well in depth and in time for $S(0) = 1$ and $M = 9$ kg/s (*a*); $M = 2$ kg/s (*b*), the Sredne-Vilyuisky field

1 – nonconjugate statement, 2 – conjugate statement

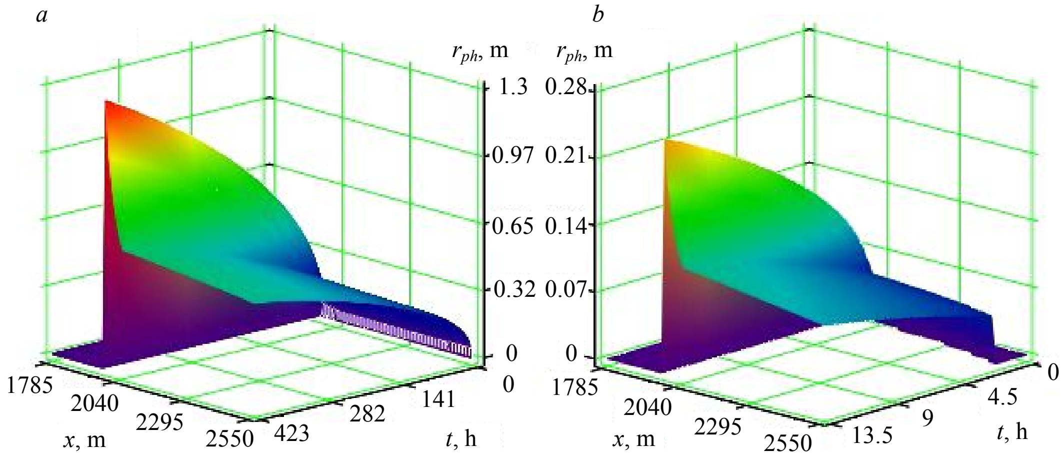

Fig.3. Change of the thawing radius around well in depth and in time for $S(0) = 1$ and $M = 9$ kg/s (*a*); $M = 2$ kg/s (*b*), the Sredne-Vilyuisky field

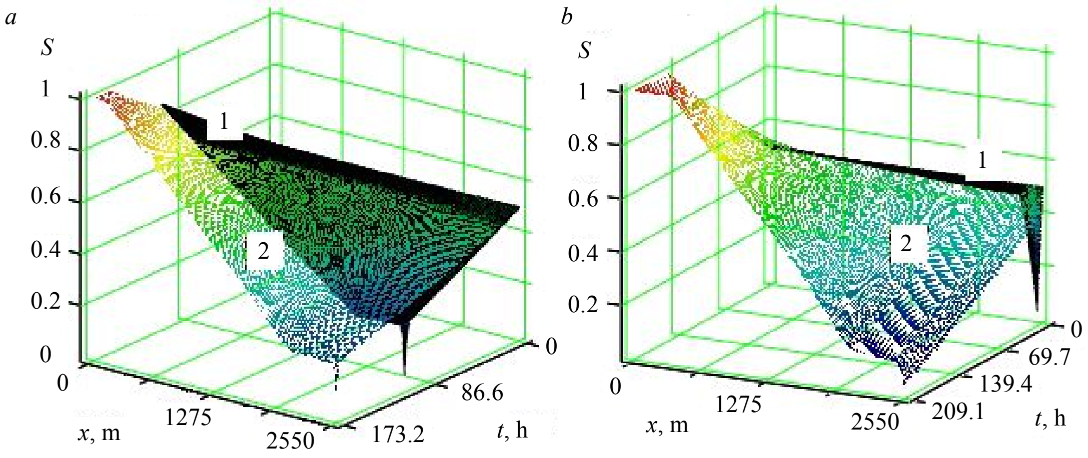

Fig.4. Change of the flow area of well in depth and in time for $S(0) = 0.5$ and $M = 4.8$ kg/s (*a*); $M = 2$ kg/s (*b*), the Sredne-Vilyuisky field

1 – nonconjugate statement, 2 – conjugate statement

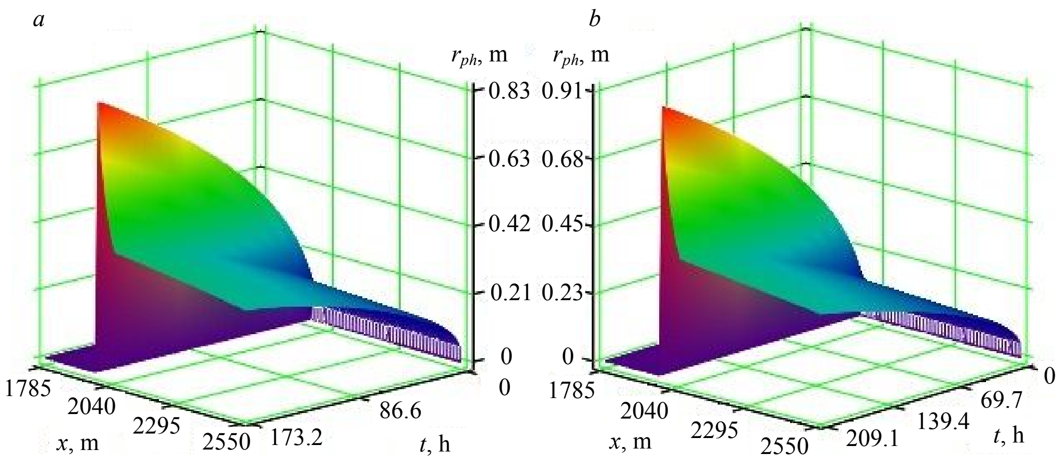

Fig.5. Change of the thawing radius around well in depth and in time at $S(0) = 0.5$ and $M = 4.8$ kg/s (*a*); $M = 2$ kg/s (*b*), the Sredne-Vilyuisky field

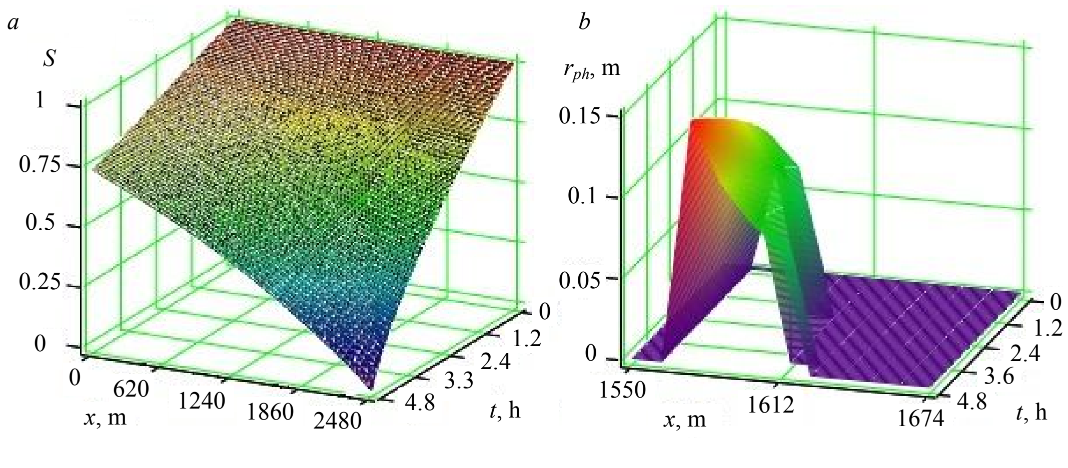

Fig.6. Change of the area of flow cross section (*a*) and the thawing radius (*b*) of the well in depth and in time for $S(0) = 1$ and $M = 2.86$ kg/s, Otradninsky field

is at depths of 56 and 52 m, for the non-conjugate and conjugate problems, correspondingly. At the same time, in the lower part of the well, from the bottom hole to the depth of 1415 m (for a non-conjugate statement) and 1160 m (for a conjugate one), the area of the cross-section increases with time, that isdecomposition of hydrates takes place. For a non-conjugated problem at the end of the gas selection process, a section at the slope level free of hydrates remains (see surface 1 in Fig.4, *a*). At a lower flow rate in the interval from the bottom hole to the depth of 1428 m (for the non-aligned problem) and 1173 m (for the conjugate one), the area of the cross-section also increases with time. Above these marks, the thickness of the hydrated layer increases with time, forming a hydrate plug near the well head in 209 h for the conjugate problem and in 9 h for the non-conjugate one (Fig.4, *b*). For a non-conjugated problem at the end of the gas production a cross section $S = 0.53$ remains free from hydrates at the bottom hole (see surface 1 in Fig.4, *b*), and for the conjugated problem part of the well from the bottom hole to 357 m is completely cleared of hydrates. Note that these marks exceed the depth at which the gas temperature becomes equal to the equilibrium temperature of hydrate formation. The depth of thawing of rocks in this case is less than for a well with a hydrate free section at the initial moment, and for these two mass flow rates it does not exceed 1 m (Fig.5, *a* and 5, *b*). This is explained by a much shorter time of thermal interaction of gas with rocks.

Now consider analogous processes for the Otradninsky field. It differs from Sredne-Vilyuisky one by low reservoir temperature which is close to equilibrium temperature of hydrate formation. From the dynamics of free cross section of the well (Fig.6, *a*) it follows that if it is free initially then at mass flow rate 2.86 kg/s the well will be completly plugged in 4.8 h, herewith a radius of rock thawing is about 0,15 m (Fig.6, *b*). The results of calculations for conjugate and non-conjugate statement are practically equivalent, that is why there are no marks for the surfaces on the Fig.6, *b*.

Modeling the operation of the main gas pipeline. Main gas pipeline «Power of Siberia», which is intended for transporting natural gas from the Chayandinsky and Kovyktinsky gas fields located in the territory of the Republic of Sakha (Yakutia) and the Irkutsk region, to the Far Eastern regions of Russia, and also to China and other countries The Asian-Pacific coast, was used as an example of modeling the process. As the initial stage the construction of the section «Chayanda-Lensk», length of 207 km and tube diameter of 1.4 m, working pressure of 9.82 MPa is planned. This area is characterized by almost continuous permafrost. To prevent undesirable consequences of its thawing the project provides for the thermal insulation of pipeline made of «Extrol-45» (extruded polystyrene foam) 0.2 m thick with a coefficient of thermal conductivity of 0.029 W/(m K). Considering the high cost of the material (about 6900 rubles/m^3), it is appropriate to assess the need for such a design solution.

In addition, the consequences of a possible deviation from the design solution, which provides for a thorough drying of the gas before it is injected into the pipeline, was considered. The reservoir conditions of the Chayandinsky field (temperature 282.15 K, pressure 13 MPa) [9] fully correspond to the equilibrium conditions of natural gas hydrates formation (its composition, %: CH_4 85.1366, C_2H_6 4.5969, C_3H_8 1.5641, iC_4H_{10} 0.5886, iC_5H_{12} 0.1734, CO_2 0.1441, N_2 7.3031, He 0.4034, H_2 0.0646, CH_3OH 0.0226. Its drying, according to the project, should bring the mole fraction of water to 0.0026 %. The maximum amount of gas produced at the Chayandinsky field should be 25 billion m^3/year, i.e. approximately 700 kg/s.

For describing the process under investigation the system (1)-(2) was used, in which the slope of the pipe to the horizontal plane was set equal to zero and the gas imperfection coefficient would be determined from the Berthelot equation in the form proposed in the monograph [6]:

$$Z = 1 + 0.07\frac{p}{p_c}\frac{T_c}{T}\left(1 - 6\frac{T_c^2}{T^2}\right).$$

Calculations were carried out with the following initial data: $D_0 = 1.4$ m, section length $L = 200000$ m, operating pressure $p_0 = 98\cdot10^5$ Pa, inlet temperature $T_0 = 282.15$ K, heat transfer coefficient taking into account thermal insulation $\alpha_0 = 0.145$ W/(m$^2\cdot$K), the initial temperature of the soil $T_{fr} = 271.15$ K; for weakly billeted sandy loam the thermal conductivity and bulk heat capacities in thawed and frozen states, respectively, $\lambda_{th} = 1.6$ W/(m·K), $\lambda_f = 1.7$ W/(m·K), $C_{th} = 2.8 \cdot 10^6$ J/(m^3·K), $C_f = 2.1 \cdot 10^6$ J/(m^3·K); density $\rho = 1760$ kg/m^3, moisture content $\omega = 0.233$. The gas constant $R = 453.524$ J/(kg·K), critical parameters $p_c = 45.01 \cdot 10^5$ Pa and $T_c = 195.075$ K,

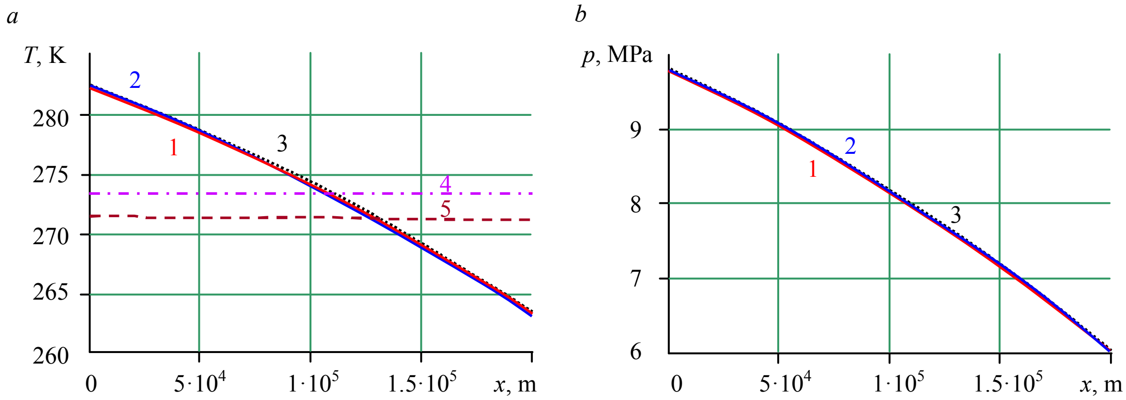

Fig.7. Distribution of temperature (a) and pressure (b) of gas along the length of a completely insulated pipeline
1 – t = 4.2 min; 2 – t = 70 h; 3 – t = 120 h; 4 – T_{ph}; 5 – T_w at t = 120 h

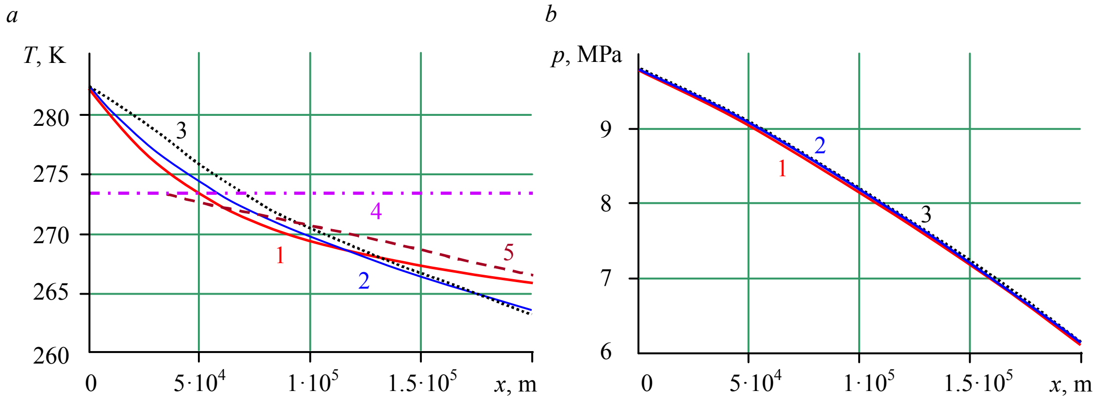

Fig.8. Distribution of temperature (a) and pressure (b) of gas along the length of a partially insulated pipeline
1 – t = 4.2 min; 2 – t = 70 h; 3 – t = 120 h; 4 – T_{ph}; 5 – T_w at t = 120 h

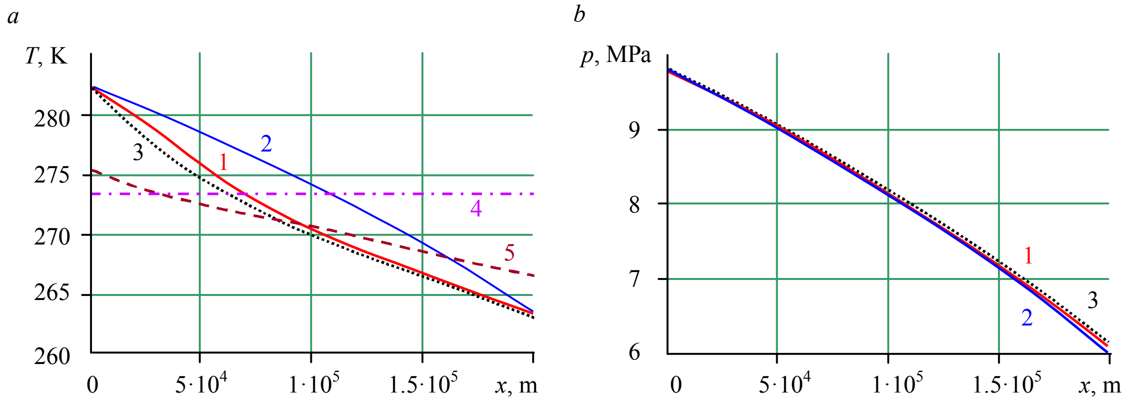

Fig.9. Distribution of temperature (a) and pressure (b) of gas along the length of the gas pipeline after 120 hours of operation for different types of thermal insulation
1 – partial; 2 – along the entire length; 3 – without insulation; 4 – T_{ph}; 5 – T_w

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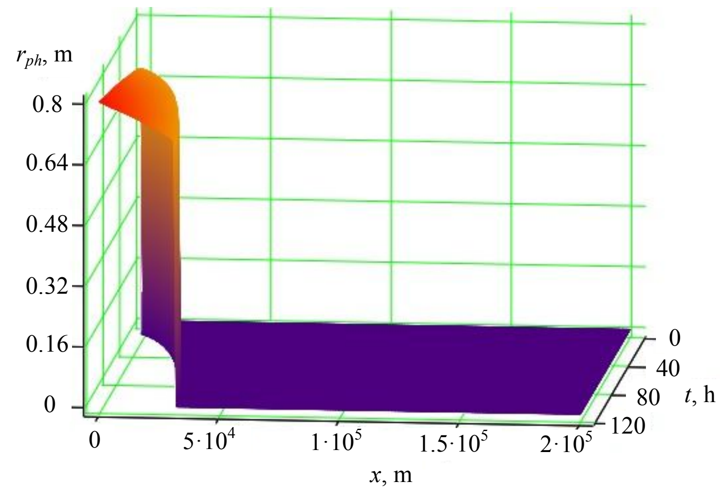

Fig.10. Dynamics of soil thawing around the gas pipeline without thermal insulation along its length

coefficients $a = 10.73$ K and $b = 117.979$ K were found for the natural gas of the Chayandinsky field. Gas mass flow rate $M = 700$ kg/s. The ground depth of K60 steel pipes with a wall thickness of 0.032 m and a thermal conductivity of 68 W/(m·K) is provided 1.5 m to the top of the pipe. The remaining parameters have the same values as in the problem of formation of hydrates in the well.

Two scenarios were realized: 1) the ideal implementation of the project, i.e. dry gas is supplied to the pipeline, which completely eliminates the formation of hydrates; 2) moist gas is supplied to pipeline. For both scenarios along with other parameters (pressure and temperature of the gas, dynamics of thawing of the ground), the dynamics of hydrate formation is determined and calculations are carried out until the outlet pressure drops below the permissible value of 4 MPa. In each of the scenarios, three variants of calculations were analyzed: 1) complete heat insulation of the pipeline; 2) thermal insulation of the initial section of the pipeline, the length of which was determined by the condition of equality of temperature at the contact of the outer surface of the pipe with the soil T_w, to the thawing and freezing temperature of the soils T_{ph}; 3) without thermal insulation.

The analysis of the calculation results begins for the first scenario (Fig.7-10). The main conclusion from the curves presented in these figures is the following: significant cooling of gas takes place due to throttling, which significantly reduces the risk of frozen ground thawing. Moreover, with a completely insulated pipeline, the gas is cooled below the freezing point of water in the ground after about 100 km, but the temperature at the contact of the gas pipeline with the ground is always below this temperature (Fig.7, *a*), i.e. the soil will not thaw. We also note that the distribution of temperature and pressure in the gas pipeline very quickly goes to the stationary regime (curves 1 in Figs.7, *a* and 7, *b*) and does not change in the future, so that curves 1, 2, 3 in these figures are practically merge. This is explained by the sharp difference in the duration of the transient processes in the gas pipeline and in the surrounding soil.

At partial thermal insulation (the first 90 km), the contact temperature T_w in 5 days becomes equal to the phase transition temperature T_{ph} only at the head section (about 30 km), and then decreases significantly (Fig.8, *a*). It means that the cost of heat insulation can be reduced by at least 2 times.

As expected, without thermal insulation the gas temperature in the initial pipeline section is slightly lower than that for partial thermal insulation, and then they are practically equal (see curves 1 and 3 in Fig.9, *a*).

In the absence of thermal insulation, the contact temperature T_w exceeds the phase transition temperature T_{ph} at the initial section of approximately 25 km (curve 5 in Fig.9, *a*). In this case, the ground thaws approximately at the same distance and the thawing radius after 5 days does not exceed 0.8 m (Fig.10). As can be seen from Fig.7, *b* – Fig.9, *b*, the temperature regime practically does not influence the pressure distribution in the pipeline.

Turning to an analysis of the results of calculations for the second scenario in which it is assumed that for some reason the pipeline receives moist gas, so that in analyzing the parameters of its transport, it is necessary to take into account the possibility of formation of gas hydrates and, consequently, formation of the hydrate layer. These results are presented in Fig.11-13. Calculations continued until the time when the outlet pressure became 4 MPa.

First of all, we note that the calculation time for the second scenario is significantly reduced, which means that the hydrate formation process is quite intensive, and in a short time (about 7 hours) the output pressure is reduced to a predetermined value. The gas temperature is everywhere below the equilibrium temperature of hydrate formation, and this difference increases with time

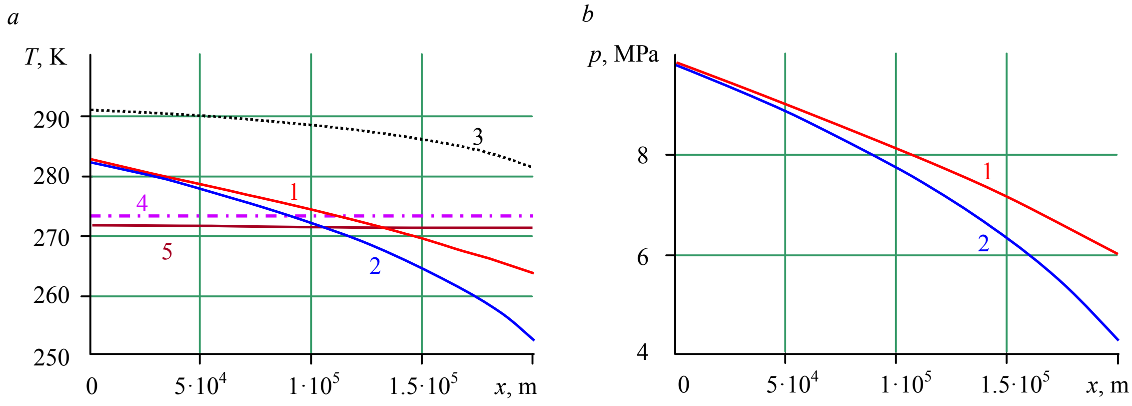

Fig.11. Distribution of temperature (*a*) and pressure (*b*) of gas along the length of a completely insulated pipeline

1 – t = 4.2 min; 2 – t = 7 h; 3 – T_h at t = 7 h; 4 – T_{ph}; 5 – T_w at t = 7 h

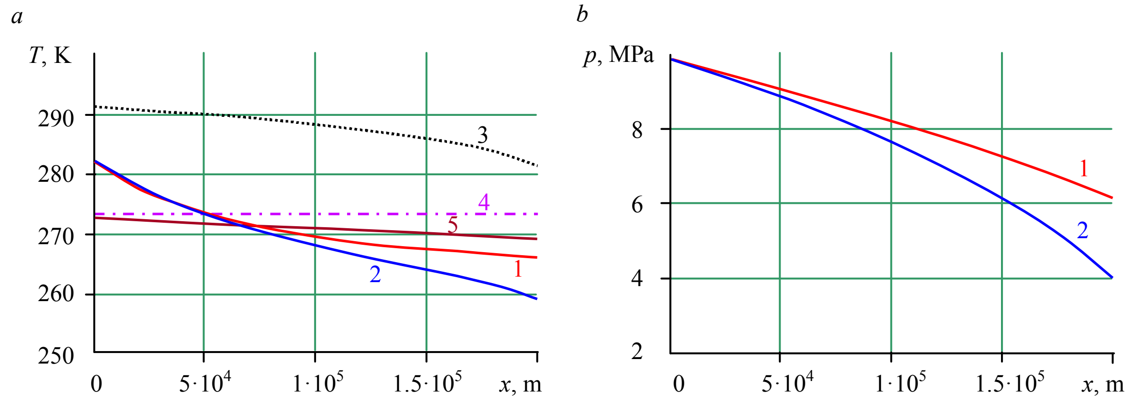

Fig.12. Distribution of temperature (*a*) and pressure (*b*) of gas along the length of pipeline without thermal insulation

1 – t = 4.2 min; 2 – t = 6,3 h; 3 – T_h at t = 6,3 h; 4 – T_{ph}; 5 – T_w at t = 6,3 h

(compare curves 1 and 2 in Figs.11, *a* and 12, *a*). In addition, in contrast to the first scenario, the pressure also decreases with time (compare curves 1 and 2 in Figs.11, *b* and 12, *b*), which is caused by a decrease in the pipe cross-section. Even more importantly, in this scenario the temperature at the contact of the pipe with the soil T_w is always below the thawing temperature T_{ph} (compare curves 4 and 5 in Figs.11, *a* and 12, *a*). Here the effect of the so-called «heat curtain» is manifested, the role of which is played by the forming hydrate layer. Because of this effect, the temperature distribution along the length of partially insulated pipeline and pipeline without insulation proves to be practically the same, and therefore only the results for the last variant of calculations are given here (Fig.12). The dynamics of the distribution of the cross-section area along the length of pipeline (Fig.13) shows that the effect of thermal insulation on the thickness of hydrate layer is ambiguous. In the presence of thermal insulation the thickness of this layer is less than when it is absent almost along the entire length (about 170 km) of the gas pipeline. However, in the final section the situation reverses (compare surfaces 1 and 2 in Fig.13).

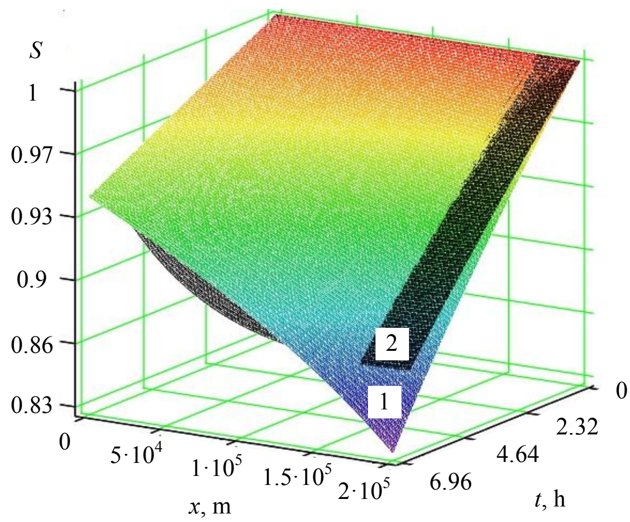

Fig.13. Dynamics of the distribution of a dimensionless area of the pipe cross section

1 – full thermal insulation; 2 – absence of thermal insulation

Conclusion. The presented results allow drawing the following important conclusions. First, the problems of thermal interaction of gas flow in wells with the surrounding rocks are generally conjugate. A simplified mathematical model in which the rock temperature is assumed to be non-variable in time leads to a significant underestimation of the time of such internal processes in wells as formation of hydrate plugs. It is shown that for deep wells with reservoir temperature significantly exceeding the equilibrium temperature of hydrate formation, this underestimation can be multiple. Secondly, the size of thawing zone of rocks indirectly depends on gas mass flow rate, because it determines the time of thermal influence of gas on the environment. In particular, for 18 days the maximum thawing radius was 1.3 m on the permafrost lower boundary. Thirdly, for deep wells with reservoir temperature approximately equal to the formation temperature of hydrates, hydrate plugs can be formed in 4-5 hours, so the time of thermal impact on rocks is short and in this case the necessary technological parameters of gas production can be determined in a non-conjugate statement.

To ensure no-failure and cost-effective operation of main gas pipeline it is necessary to predict and systematically monitor its thermal regime, which is a part of the overall system of monitoring of various parts of the whole object. The reliability of such systems is especially important for main gas pipelines operating in complex natural-climatic, geocryological and hydrological conditions, as is the case of the projected «Power of Siberia» gas pipeline.

The results of the computational experiment make it possible: 1) to assess the risk of emergency pressure reduction at insufficient gas drying; 2) reduce the cost of heat insulation of gas pipelines.

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