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# MULTIPHYSICAL MODEL OF HETEROGENOUS FLOW MOVING ALONG A CHANNEL OF VARIABLE CROSS-SECTION

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The article deals with the problem aimed at solving the fundamental problems of developing effective methods and tools for designing, controlling and managing the stream of fluid flowing in variable-section pipelines intended for the production of pumping equipment, medical devices and used in such areas of industry as mining, chemical, food production, etc. Execution of simulation modelling of flow motion according to the scheme of twisted paddle static mixer allows to estimate the efficiency of mixing by calculating the trajectory and velocities of the suspended particles going through the mixer, and also to estimate the pressure drop on the hydraulic flow resistance. The model examines the mixing of solids dissolved in a liquid at room temperature. To visualize the process of distributing the mixture particles over the cross-section and analyzing the mixing efficiency, the Poincaréplot module of the COMSOL Multiphysics software environment was used.

For the first time, a multi-physical stream of heterogeneous flow model has been developed that describes in detail the physical state of the fluid at all points of the considered section at the initial time, takes into account the design parameters of the channel (orientation, dimensions, material, etc.), specifies the laws of variation of the parameters at the boundaries of the calculated section in conditions of the wave change in the internal section of the working chamber-channel of the inductive peristaltic pumping unit under the influence of the energy of the magnetic field.

***Key words*:** heterogeneous flow, mathematical modelling, elastomer, mixer, peristaltic transportation

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**Introduction.** Many problems of hydromechanics, hydraulics and the organization of technological processes due to the moderate velocities of medium displacement can be studied within the framework of model of an incompressible viscous fluid [1]. At the present time, there are various types of problems which complete investigation can be carried out only by means of a computational experiment or an accurately set up physical experiment [2, 3, 5, 10, 11, 14, 15]. However, phenomena and technological processes having a practical interest are often not available for comprehensive physical modeling and the costs of such experiments are excessively large.

The issues having practical interest, as a rule, are characterized by multidimensionality, nonstationarity, nonlinearity, the presence of free boundaries and boundary layers, and are described by the Navier – Stokes equations [6-9, 12, 13]. The nonlinearity of the Navier – Stokes equations and the presence of a small parameter for the highest derivatives (especially for large Reynolds numbers) create serious difficulties both in their analytical investigation and in the numerical solution of these equations by means of a computer.

**Formulation of the problem.** The inductive peristaltic pumping unit is designed for movement of fluid with the help of a wave of local deformation of the working chamber-channel of incompressible materials through a pipe of variable geometry.

The working chamber-channel of inductive peristaltic pumping unit plays a role of a static mixer. The transported material is pumped through a pipe with stationary screws. This mixing method is especially efficient for mixing with laminar flow since it creates only small losses of flow pressure (Fig.1).

The peristaltic pipeline can be viewed as a deformable body along which the forced waves of contraction and elongation deformation move. There is no transfer of pipeline particles during this process. Part of the sections of the pipeline is compressed, reducing the cross-sectional area, and some of the subsequent sections are expanded. This process sequentially goes along the pipeline,

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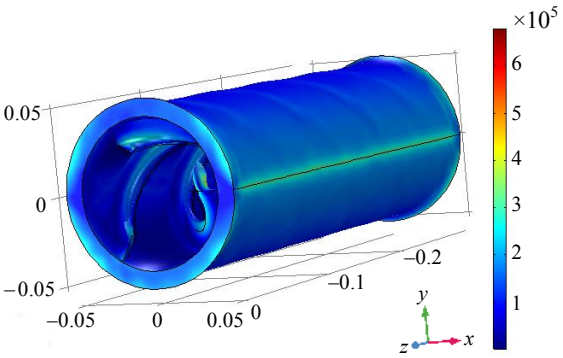



Fig.1. Internal surface relief pattern of working chamber-channel

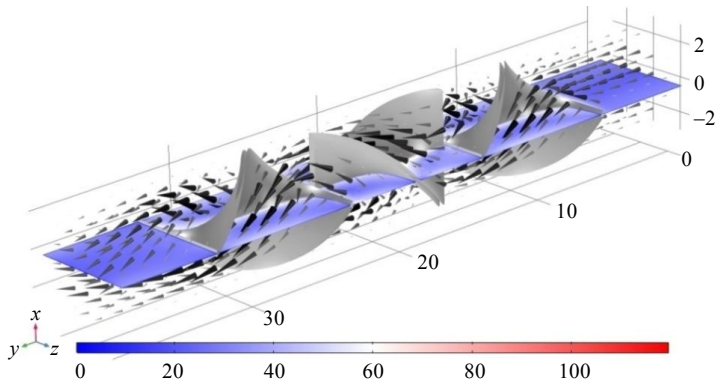



Fig.2. Distribution of velocity vectors when passing of a section of a working chamber-channel of a mixer

forming a moving wave. The compression of the pipe allows to pump the mixture using a laminar flow with a relatively low shear stress. The relief of the internal surface of the pipeline ensures the structuring of the flow, as well as its mixing simultaneously with transportation, which prevents the delamination of non-homogeneous media [16, 17].

**Methodology.** When the media streams around the body along with head resistance there may appear the components of viscous forces, which are orthogonal to the velocity vector of the body relative to the moving media (the lifting force according to Zhukovsky).

Viscosity is a macroscopic manifestation of molecular motion and mixing. If macroscopic pulsations are formed in the fluid, this leads to mixing on a macroscopic scale and to changing the viscous forces, in particular, the dragging forces with which the flow acts on the streamlined body and the walls of the tubes. Therefore, the equation given for the pressure field $p(\tau,t)$ and velocity field $u(\tau,t)t$

$$p\left[\frac{\nabla u(\tau,t)}{\nabla t}+(u(\tau,t),\nabla)u\right]=-\nabla p-\eta\nabla u(\tau,t), \tag{1}$$

is valid only if the velocity does not exceed the critical value determined by the Reynolds number, i.e. the current lines of real currents do not become «tortuous».

The Prandtl formula for turbulent viscosity:

$$\eta_T=l^2\left|\frac{\nabla u}{\nabla y}\right|, \tag{2}$$

where $l$ – mixing length determined by experiments; $u$ – average value of flow velocity.

Simulation modeling of flow motion according to the scheme of a twisted paddle static mixer allows estimating the mixing efficiency by calculating the trajectory and velocities of the suspended particles through the mixer, and also estimating pressure losses due to the hydraulic flow resistance (Fig.2).

The model examines the mixing of solids dissolved in a liquid at room temperature. The geometry of the channel consists of a tube with three twisted screws of variable rotation.

The flow at the inlet is laminar and fully developed at an average speed of 1 cm/s. At the end of the process, the model defines a constant head pressure of 0 Pa. Laminar flow is described by the following equation

$$p(u\nabla)u=\nabla\left[-pI+\mu\left(\nabla u+(\nabla u)^T\right)\right], \tag{3}$$

$$\nabla u=0.$$

The particle trajectories are calculated from the Newtonian model using the Stokes drag force law [4]:

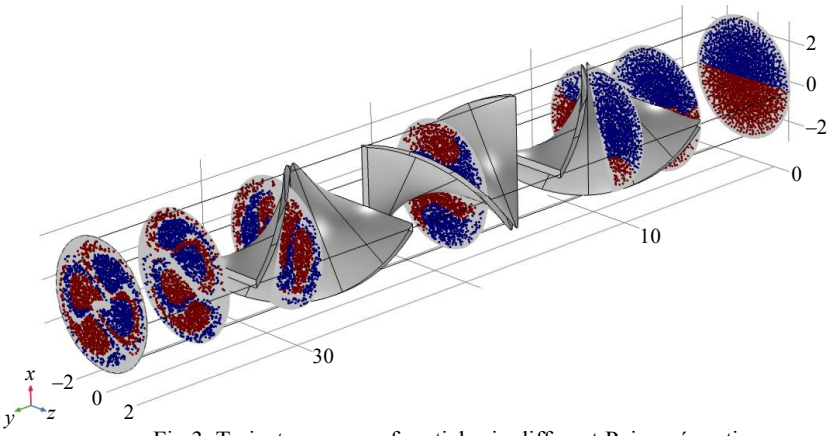



Fig.3. Trajectory maps of particles in different Poincaré sections

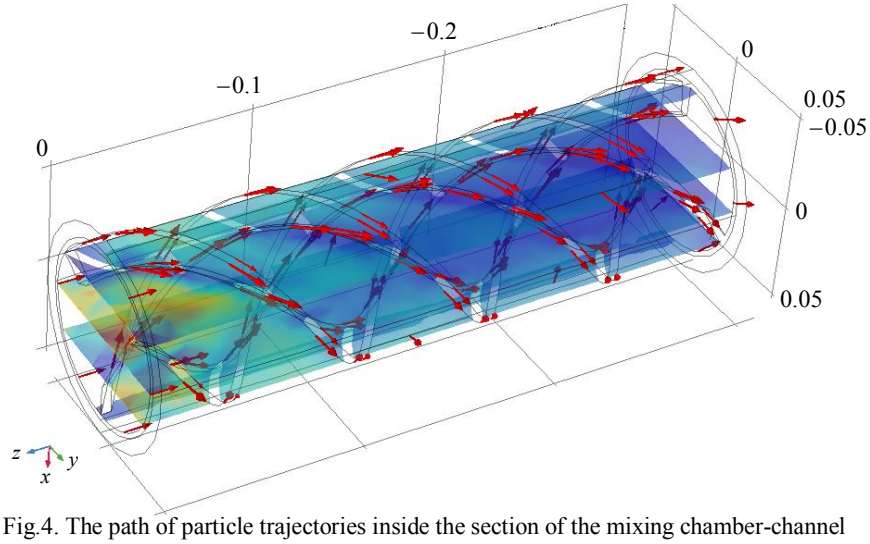



Fig.4. The path of particle trajectories inside the section of the mixing chamber-channel

$$\frac{d}{dt}(m_p v) = \frac{1}{\tau_p} m_p (u - v), \quad (4)$$

where $v$ – particle velocity; $\tau_p$ – particle velocity response time,

$$\tau_p = \frac{\rho_p d_p^2}{18\mu}; \quad (5)$$

$\rho_p$ – density; $d_p$ – particle diameter.

The particle density is normalized in accordance with the velocity of the liquid at the input. This means that the particles are larger at the entrance to the mixer, where the speed is the highest, and smaller, where the speed is small.

**Discussion.** Since the particles have mass, they do not necessarily all reach the outlet; some particles get stuck in the wall of the working chamber-channel of the mixer. The probability of transmission is defined as the ratio of the number of particles that reach the output to the total number of particles. For this particular configuration, the transmission probability is about 0.80. This means that about 20 % of the particles remain trapped in a mixer.

Modeling the stream of heterogeneous flow allows visualizing the distribution of the solid phase and evaluating the efficiency of the mixer.

To visualize the process of distribution of the particles in the mixture over the cross-section and to analyze the efficiency of mixing, the Poincaré plot module of the COMSOL Multiphysics software environment was used (Fig.3). Poincaré plot sets a color point for each of the particles at the place where the particle passes through the cross-section plane (the Poincaré section).

The color indicates the location of the particle in its original position. Thus, the particles marked in red had an initial position $x < 0$. The particles marked in blue had an initial position $x > 0$. The operator $at(x)$ is used to designate the particles in their initial position. The first Poincaré section (rightmost in Fig.3) shows where the particles with coordinates $x < 0$ are placed. As the particles start to follow the stream, they begin to mix. By the end of the working chamber-channel, the particles are not completely mixed, there are still significant focal areas where only red and only blue particles are concentrated.

Based on the known properties of the chamber-channel material and the parameters of the transported medium, the model makes it possible to evaluate the expanding forces on the pipe walls, as well as the force necessary to move the portion of the substance (Fig.4).

The sum of all the forces acting on the impurity in the chamber-channel is

$$F = (\rho_T g - \rho_l g - \delta B), \quad (6)$$

where $\rho_T$ – density of mechanical impurities; $\rho_l$ – liquid density; $g$ – free fall acceleration; $\delta$ – current density; $B$ – magnetic field induction.

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Let us imagine that in some coordinate system the electromagnetic field is characterized by the vectors of the electric field strength and the magnetic induction $B$ of the magnetic field. In relation to this coordinate system, a particle with charge $q$ moves in the gap of the magnetic device with a velocity $v$. From the side of the electric and magnetic fields, the following force acts on this particle

$$f = qE + q[vB], \tag{7}$$

where $qE$ – a force of electric field; $q[vB]$ – a force of magnetic field.

The particle will be acted upon by electric $f_e$ and magnetic $f_m$ forces, which coincide in direction. In addition to the forces $f_e$ and $f_m$, the flow force $f_p$ of the fluid system moving along the direction of velocity $v$ will also act on the particle, as well as the force $f_{em}$ from the interaction of $B_p$ and the current flowing in the aqueous medium. Thus, the resultant force will be

$$f = f_e + f_m + f_p + f_{em}. \tag{8}$$

Under the action of these forces, the particle in the gap of the working chamber-channel will move along a spiral around the magnetic apparatus. For particles having the same charge sign (+), the direction of the spiral-like rotation will be the same, for particles of charge (–), the direction of the spiral-shaped rotation will be opposite to the particles of the sign (+).

**Conclusion.** The developed multi-physical model of the stream of heterogeneous flow allows detailed description of the physical state of the substance at all points of the considered volume at specific instants of time, takes into account the design parameters of the channel (orientation, dimensions, material, etc.), and also specifies the laws of variation of parameters at the boundaries of the calculated region in conditions of the wave change of the internal section of the working chamber-channel under the influence of the energy of the magnetic field.

The significance of the results lies in the study of the effect of the factors that complicate calculation, namely the nonstationarity of the three-dimensional flow process in the channel with changing geometry, the presence of volume forces, the heterogeneity of the flow, the dependence of the thermophysical properties on the flow state parameters, the influence of the internal viscosity of the flow, possible phase transitions, various hydrodynamic flow regimes, turbulence, influence of magnetic fields, stress arising in the pipe material.

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