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3D-DIAGNOSTICS OF FUNCTION OF ELECTRON DISTRIBUTION IN PLASMA

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The paper gives further development of the method of a plate single-sided probe, which makes it possible to reconstruct the total electron velocity distribution function in an axially symmetric nonequilibrium plasma with an arbitrary degree of anisotropy. The method is improved for plasma diagnostics without the assumption of any symmetry. The theory of the method is developed and analytical relations are obtained connecting the Legendre components of the second-order derivative of the probe current with respect to the potential of the probe and the electron distribution function. The method is experimentally tested in the plasma of a positive column of a helium glow discharge. New possibilities of the method for investigating plasma near the boundaries are demonstrated and non-traditional information is obtained on the processes of escape of charged particles from the plasma volume on the walls.

Key words: distribution function of electrons, plate probe, radial electrical field, loss cone, Langmuir paradox

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Introduction. One of the most important fundamental problems in the physics of gas-discharge plasma is the «Langmuir paradox», that has been known since the first half of the last century [9] and widely discussed in the literature to this day [2, 4, 6, 7, 9].

In his paper L.D. Tsendin [13] attempted to explain the Langmuir paradox in a low-pressure plasma. As a result, it was concluded that the main mechanism that determines the form of the velocity distribution function of electrons (VDFE) is the escape of electrons from the plasma volume, which is determined by the elastic scattering of electrons in the loss cone.

The first systematic experimental investigations of the VDFE under the conditions of the Langmuir paradox existence were carried out by Yu.M. Kagan and his co-workers [3]. Their result was a conclusion about the unknown «wall» mechanism of VDFE maxwellization process.

In 1968 S.W. Rayment and N.D. Twiddy [12] presented the results of a series of experiments in which spherical and cylindrical probes located perpendicular to the axis of the tube were used. With this orientation, the cylindrical probe does not «feel» the loss cone, and the registered VDFE has a Maxwellian form in a wide range of energies. When measuring with a cylindrical probe located at the center of the tube parallel to the axis of the discharge, a strong depletion of the second-order derivative of the probe current was found with the potential of the probe located above the wall probe. Most clearly, such depletion, associated with the escape of electrons, was observed during measurements by a plate wall probe [12]. In [11], there were also differences in the recorded second-degree derivatives at two orientations of the cylindrical probe – along and across the discharge axis.

The first targeted researches of electron loss cone were performed by the authors of paper [8]. The experiment was conducted in high frequency discharge in argon. The electron distribution function was registered with the help of plate probe located at 1 cm distance from the wall. It has been determined that with increase of electron energy the direction diagrams start to have peculiar features related to depletion of VDFE due to escape of electrons from plasma volume.

In order to register the loss cone and advance in understanding of Langmuir paradox we need to have new reliable diagnostics methods enabling to record VDFE with angular resolution near the boundaries of plasma volume, to measure radial profiles of electric field, wall jumps of potential, electron concentration, etc.

To solve this problem, we proposed and experimentally tested a new probe method for plasma diagnostics with an arbitrary degree of symmetry. The first experimental results are obtained, which are in good agreement with the conclusions of [8, 13].

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Experimental procedure and method. To conduct the experimental research, we have used an installation shown in Fig.1. The layout of the installation is divided into four blocks, the elements of each block have their own numbering scheme.

The positive glow (block I) was created in quartz tube with a diameter of 3 cm and length of 30 cm between plate impregnated separately heated cathode with a diameter of 11 mm and molybdenum anode with a diameter of 20 mm. Block II consists of vacuum chamber with internal diameter of 160 mm and length of 0.5 m and sapphire windows with a diameter of 40 mm designed for observation of discharge glow and conducting optic researches.

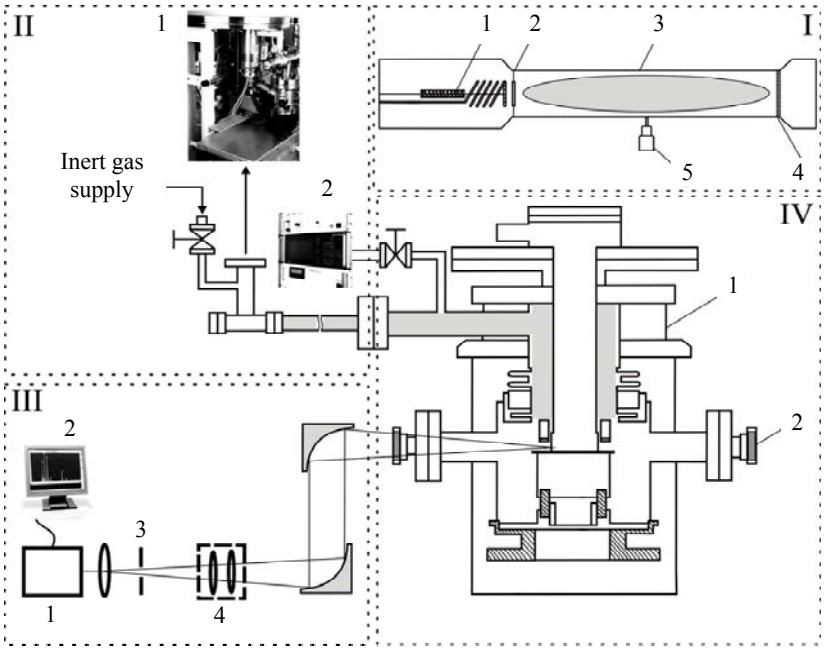


Fig.1. Principal diagram of experimental installation
Block I – a tube for creating positive glow (1 – heater, 2 – cathode, 3 – quartz tube, 4 – anode, 5 – probe); block II – elements of vacuum system (1 – turbomolecular pump, 2 – complex of mass spectrometric analysis); block III – optical block (1 – monochromator, 2 – system for processing experimental data, 3 – diaphragm, 4 – condenser); block IV – elements of vacuum system (1 – vacuum chamber, 2 – sapphire window)

Block IV shows elements of vacuum system. Vacuum was created by turbomolecular pump, the limited underpressure is $5 \cdot 10^{-8}$ Torr. The fluent inflow of spectral pure gas in the chamber was done with the help of a system of reducers and needle inlet valve. In the present experiment, we used helium, which spectral purity was controlled by a specialized complex of mass spectrometric analysis in conjunction with the optical diagnostic system shown in block III.

The probe measurements were done with plate single-sided probes, that were produced using special technology. With the help of punch and matrix made of tantalum foil 30 μm thick, probes of 0.5 mm in diameter were punched. The holders made of tantalum foil with a diameter of 0.1 mm were attached to probes by sport welding. One of the sides of the probe together with a holder were covered with alum mastic, and later the isolating layer was sintered in vacuum at a temperature of 1800 K.

The probes were mounted on a three-coordinate micrometric remote movement system, which with the help of bellows joint provided their spatial positioning with an accuracy of ±0.01 mm and orientation in respect to charge symmetry axis with an accuracy not less than ±10 s. The monitoring of probe positioning was done by eyepiece micrometer.

The experiment was monitored by a multichannel measurement and calculation system based on a PC. The special software and radio-technical element base allowed to perform digital data recording that were received in pulsed and stationary modes, as well as their complex software processing in real time scale regime.

The method of measuring the second-order derivative of the probe current with respect to the potential of the probe was described in papers [4, 6]. We have used a modulating signal having the form of $u(t) = \Delta\varepsilon(1 + \cos\omega_1 t)\cos\omega_2 t$. The consideration of hardware distortions, selection of optimal amplitude of differentiating signal and control of linearity of recording system were carried out according to the method described in [13]. We used the following values $\Delta\varepsilon = 0.2$ B, $\omega_1 = 6 \cdot 10^3$ Hz, $\omega_2 = 6 \cdot 10^5$ Hz.

All the factors influencing the accuracy of probe measurements [13] were carefully taken into account in the formulation of the experiment, which ensured the reproducibility of the experimental results no worse than 0.5%.

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Development of method of 3D plasma diagnostics with random symmetry. ***Fundamentals of plate probe method for diagnosing axially symmetric plasma.*** Currently the method of plate single-sided probe [5, 11] enables obtaining the most complete information about properties of non-equilibrium anisotropic plasma. This method is a development of a traditional method of Langmuir probes [10] and is designed to diagnose a plasma with an arbitrary degree of anisotropy. It was developed for the case of axially symmetric stationary plasma and allows to reconstruct the complete electron and ion distribution function in terms of velocity and angles (Fig.2). In such a plasma in spherical coordinate system with a polar axis directed along the symmetry axis the VDFE with respect to velocities does not depend on the azimuthal angle:

$$f(\vec{r},\vec{v}) = f(\vec{r},v,\theta), \tag{1}$$

where $v = |\vec{v}|$; θ – polar angle.

Hereafter for the sake of brevity we will omit the spatial variable. Let us give a short derivation pf the basic equations of this method. The electron current to the probe from the plasma under the assumption that all electrons that overcome the potential barrier of the probe are absorbed by it and that there is no secondary emission from its surface,

$$I = qS\int v_n\, f(\vec{v})d\vec{v} = \frac{2qS}{m^2}\int_0^{2\pi} d\varphi' \int_{qU}^{\infty} \varepsilon d\varepsilon \int_0^{\theta'_{\max}} f(\varepsilon,\theta',\varphi')\cos\theta'\sin\theta' d\theta', \tag{2}$$

where v_n – normal to the probe surface component of electron velocity vector $\vec{v}$, $v_n = v\cos\theta' \ge v_{\min} = (2qU/m)^{1/2}$; $\varepsilon = mv^2/2$; φ' *and* θ' – azimuth and polar angles of vector $\vec{v}$ in a spherical coordinate system, which polar axis matches the normal line to the non-conducting surface of the plate probe; U – negative potential in relation to plasma.

Differentiating (2) twice with respect to potential U, we obtain

$$I''_U = \frac{q^3 S}{m^2}\left[\int_0^{2\pi} f(qU,\theta'=0,\varphi')d\varphi' - \int_0^{2\pi} d\varphi' \int_{qU}^{\infty} \frac{\partial}{\partial(qU)} f(\varepsilon,\theta'_{\max},\varphi')d\varepsilon\right]. \tag{3}$$

Let us shift to (3) in laboratory coordinate system, in which VDFE has a form (1). To do this we will use the relation connecting a polar angle of laboratory coordinate system θ with angles θ' and φ' and an angle α between polar axes of coordinate system:

$$\cos\theta = \cos\theta'\cos\alpha + \sin\theta'\sin\alpha\cos\varphi'.$$

Then we come to the expression

$$I''_U(qU,\alpha) = \frac{2\pi q^3 S}{m^2}\left[f(qU,\alpha) - \frac{1}{2\pi}\int_0^{2\pi} d\varphi' \int_{qU}^{\infty} \frac{\partial}{\partial(qU)} f(\varepsilon,\theta^*)d\varepsilon\right]. \tag{4}$$

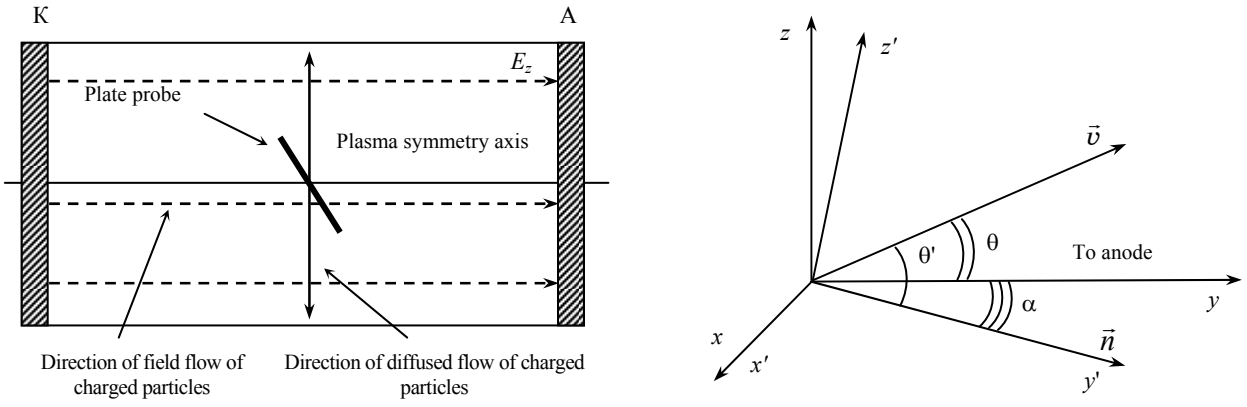


Fig.2. Geometry of the task in case of axial symmetry

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The relation (4) is a very complex integral equation with respect to the desired VDFE. To find from it the distribution function let us formularize $f(\varepsilon, \theta)$ and $I''_U(qU, \alpha)$ in the form of expansion in series of Legendre polynomials [10, 11]. We note that these series converge absolutely and uniformly, if the functions f and I''_U are twice differentiable with respect to the angle:

$$f(\varepsilon, \theta) = \sum_{j=0}^{\infty} f_j(\varepsilon) P_j(\cos\theta); \tag{5}$$

$$I''_U(qU, \alpha) = \frac{2\pi q^3 S}{m^2} \sum_{j=0}^{\infty} F_j(qU) P_j(\cos\alpha). \tag{6}$$

It should be noted that Legendre components $f_j(\varepsilon)$ have a separate physical meaning and define a set of important plasma parameters. For example, a component f_0 defines the plasma concentration:

$$n = \frac{4\pi\sqrt{2}}{m^{3/2}} \int_0^{\infty} \sqrt{\varepsilon} f_0(\varepsilon) d\varepsilon; \tag{7}$$

component f_1 – density of electron flow in plasma:

$$j_e = \frac{8\pi q}{3m^2} \int_0^{\infty} \varepsilon f_1(\varepsilon) d\varepsilon; \tag{8}$$

variable f_2 – anisotropic part of the electron momentum flux density tensor:

$$\mathrm{P}_{ij} = m\int v_i v_j f(\vec{v}) d\vec{v} = \begin{pmatrix} P_0 - \frac{1}{5}P_2 & 0 & 0 \\ 0 & P_0 - \frac{1}{5}P_2 & 0 \\ 0 & 0 & P_0 + \frac{2}{5}P_2 \end{pmatrix}. \tag{9}$$

Here the scalar electron pressure is

$$P_0 = \frac{8\pi\sqrt{2}}{3m^{3/2}} \int_0^{\infty} \varepsilon^{3/2} f_0(\varepsilon) d\varepsilon,$$

anisotropic component of pressure

$$P_2 = \frac{8\pi\sqrt{2}}{3m^{3/2}} \int_0^{\infty} \varepsilon^{3/2} f_2(\varepsilon) d\varepsilon.$$

The remaining components, along with the first three, determine the angular structure of the VDFE and are of particular interest in studying the population of the Zeeman sublevels of the atom and the scattering diagrams of electrons in collision events.

After substituting (5) and (6) in (4) we will obtain a relation between components f_j and F_j:

$$f_j(qU) = F_j(qU) + \int_{qU}^{\infty} f_j(\varepsilon) \frac{\partial}{\partial(qU)} P_j\left(\sqrt{\frac{qU}{\varepsilon}}\right) d\varepsilon. \tag{10}$$

Here we use the addition theorem for the Legendre polynomials:

$$P_j(\cos\theta^*) = P_j(\cos\alpha) P_j\left(\sqrt{\frac{qU}{\varepsilon}}\right) + 2\sum_{m=1}^{j} \frac{(j-m)!}{(j+m)!} P_j^m(\cos\alpha) P_j^m\left(\sqrt{\frac{qU}{\varepsilon}}\right) \cos m\varphi'.$$

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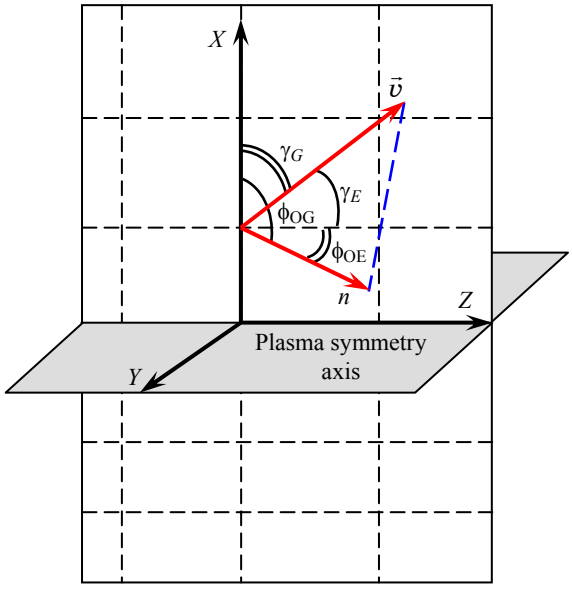


Fig.3. Geometry of the task for the probe located in radial area of plasma

Expression (10) represents the Volterra integral equation of the II kind [1]. Using its resolvent, expression (10) can be solved with respect to the Legendrian components f_j:

$$f_j(qU) = F_j(qU) + \int_{qU}^{\infty} F_j(\varepsilon) R_j(qU, \varepsilon) d\varepsilon, \qquad (11)$$

where

$$R_j(qU, \varepsilon) = \frac{2^{-(j+1)}}{qU} \sum_{k=0}^{[j/2]} a_{kj} \left(\frac{\varepsilon}{qU} \right)^{\frac{j-2k-1}{2}};$$

$$a_{kj} = (-1)^k \frac{(2j-2k)!(j-2k)}{k!(j-k)!(j-2k)!};$$

$$\left[\frac{j}{2} \right] = \begin{cases} \frac{j-1}{2} & \text{for odd } j; \\ \frac{j}{2} & \text{for even } j. \end{cases}$$

The substitution in (11) the relation for Legendre component

$$F_j(qU) = \frac{(2j+1)m^2}{4\pi q^3 S} \int_{-1}^{1} I''_U(qU, x) P_j(x) dx$$

leads to the main formula of the method

$$f_j(qU) = \frac{(2j+1)m^2}{4\pi q^3 S} \int_{-1}^{1} \left[I''_U(qU, x) + \int_{qU}^{\infty} I''_U(\varepsilon, x) R_j(qU, \varepsilon) d\varepsilon \right] P_j(x) dx. \qquad (12)$$

In conclusion we give an explicit form of the first three Legendre components:

$$f_0(qU) = \frac{m^2}{4\pi q^3 S} \int_{-1}^{1} I''_U(qU, x) dx;$$

$$f_1(qU) = \frac{3m^2}{4\pi q^3 S} \int_{-1}^{1} \left[I''_U(qU, x) + \frac{1}{2qU} \int_{qU}^{\infty} I''_U(\varepsilon, x) d\varepsilon \right] x\, dx;$$

$$f_2(qU) = \frac{5m^2}{8\pi q^3 S} \int_{-1}^{1} \left[I''_U(qU, x) + \frac{3}{2(qU)^{3/2}} \int_{qU}^{\infty} \sqrt{\varepsilon} I''_U(\varepsilon, x) d\varepsilon \right] (3x^2 - 1) dx.$$

Thus, the method of a plate single-sided probe consists in measuring the values $I''_U(qU, \alpha)$ and the subsequent calculation according to formulas (5) and (12) of the DFE with respect to velocities, as well as its Legendrian components.

Diagnosis of plasma with an arbitrary degree of symmetry. The application of the plate probe method is limited by the requirement of axial symmetry of the plasma object. Figure 3 shows the case when the probe is located in the radial region of the discharge gap and it becomes necessary to take into account the asymmetry due to the presence of a radial electric field.

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A method for diagnosing a plasma with an arbitrary degree of symmetry is proposed in this paper. In this case, the basic equations of the plate-probe method (5), (6) are transformed into the following relations:

$$I_u''(qU) = \frac{2\pi q^3 S}{m^2} \sum_{j=0}^{\infty} \{F_{jE}(qU) P_j(\cos\phi_{0E}) + F_{jG}(qU) P_j(\cos\phi_{0G})\}; \quad (13)$$

$$f_E(\varepsilon) = \sum_{j=0}^{\infty} f_{jE}(\varepsilon) P_j(\cos\phi_{0E}); \quad (14)$$

$$f_G(\varepsilon) = \sum_{j=0}^{\infty} f_{jG}(\varepsilon) P_j(\cos\phi_{0G}), \quad (15)$$

where ϕ_{0E}, ϕ_{0G} – angles between normal line to the surface of plate single-sided probe and directions of field and diffusion flow of charged particles respectively.

In the formal mathematical plane, the structure of equations (13), (14) and (15) is analogous to (5) and (6), but the number of defined components is now 2 times larger.

As in the theory of the plate probe method, the reconstruction of the components f_{jE}, f_{jG} is related to the solution of the corresponding Volterra equations connecting the Legendre components of the VDFE and the second derivative:

$$f_{jE}(qU) = F_{jE}(qU) + \int_{qU}^{\infty} F_{jE}(\varepsilon) R_j(qU, \varepsilon) d\varepsilon; \quad (16)$$

$$f_{jG}(qU) = F_{jG}(qU) + \int_{qU}^{\infty} F_{jG}(\varepsilon) R_j(qU, \varepsilon) d\varepsilon. \quad (17)$$

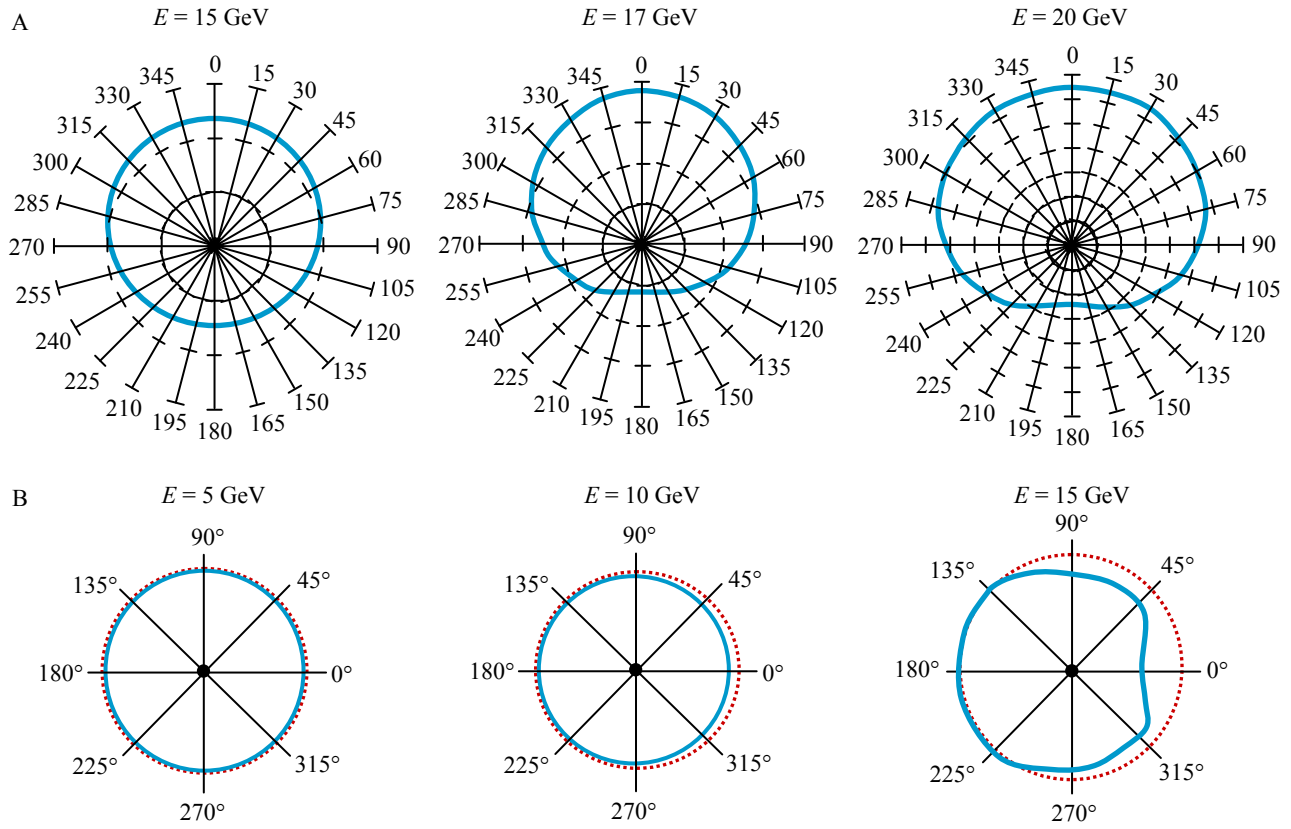


Fig.4. VDFE angle structure:
A – registered at distance of 0.2 cm from wall in the plasma of positive glow in helium
(P_{He} = 0.5 Torr, I = 0.5 A; angle 180° – collecting surface of the probe is turned to the wall)
B – registered in high-frequency discharge in argon by the authors [8]
(angle 0° – collecting surface of the probe is turned to the wall)

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Experimental verification of the plate probe method for plasma diagnostics with an arbitrary degree of symmetry. The probe measurements were carried out in the plasma of a positive helium glow discharge in the pressure range 0.1-1 Torr and discharge currents of 0.1-0.5 A at a distance of 0.2 cm from the wall of the discharge tube at two orientations of the flat probe relative to the discharge axis (0° and 180°). The values of the second-order derivative are registered, the angular structure of the VDFE in the loss cone is restored, and polar diagrams of the directed motion of the electrons are constructed (Fig.4).

As in [8], with an increase in the electron energy in the diagrams, the high-energy part of the VDFE is depleted, it is associated with the escape of electrons from the plasma to the walls. If the potential of the probe exceeds the potential of the wall, then the probe registers the VFDE in the loss cone. With increasing current and pressure, the picture does not change significantly. Rotation of the probe by 180 ° makes it possible to measure the VDFE in the main part of the plasma volume. In this case, a pattern of a weak anisotropic VDFE is seen in the directional diagrams typical of a positive glow.

Conclusion. The obtained data are in good agreement with the theoretical and experimental results [8, 13], which indicates the accuracy of the developed method and can serve as confirmation that the Langmuir paradox is associated not with the hypothetical mechanism of the VDFE maxwellization process but with the physical features of its formation as a result of a combination of already known mechanisms .

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