Non-destructive testing of multilayer medium by the method of velocity of elastic waves hodograph

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The method of velocity of elastic waves hodograph, aimed at non-destructive testing of structurally heterogeneous composite materials and products based on them, as well as multilayer products and constructions, is considered.

The theoretical basis for determining the propagation velocity of elastic waves in a multilayer medium by the hodograph method is given. Based on the studies, recommendations are given for determining the propagation velocity of elastic waves in each individual layer of a multilayer medium, which allows non-destructive testing of the physicomechanical characteristics of each layer of a multilayer medium.

It is shown that in addition to simple multiple reflections in a homogeneous medium, in a multilayer medium with parallel interfaces consisting of two or more layers, complex types of multiple reflected waves and mixed waves (reflected-refracted and refracted-reflected) can arise.

The main task of applying the low-frequency ultrasonic method is to determine the acoustic parameters of the propagation of elastic waves (velocities, amplitudes, spectra). The main methods for determining the elastic wave velocities are considered, based on the hodograph equation of the indicated reflected waves in a multilayer medium.

Key words: multilayer medium; velocity of elastic waves hodograph; reflected waves; refracted waves

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Introduction. The use of modern high-frequency ultrasonic methods for non-destructive testing of single-layer and multi-layer products of thick-walled large-sized mediums and structures, including composite and large-structural materials (concrete, asphalt concrete, wood, rocks, polymers, elastomers, foam and porous materials, etc.) is very difficult, and more often it is not possible due to the high scattering and attenuation of high-frequency elastic waves [5, 6, 9, 10, 12-14]. This is due to the fact that the wavelength of elastic vibrations (0.1-3 mm) is commensurate with the dimensions of the structural elements of these materials. The reflection of elastic waves on inclusions creates complex phenomena of interaction of elastic waves due to their reverberation, interference, and diffraction. Control of these materials by this method becomes possible only when the wavelength of elastic vibrations is an order of magnitude greater than the predominant size of the inclusions. The fulfillment of this condition is possible in the case of using the frequency range in the region of low ultrasonic frequencies – in the range of 20-200 kHz [5].

The low-frequency acoustic methods of non-destructive testing of multilayer mediums [5], the length of which in the direction of wave propagation is several times greater than the thickness of the layers, became widespread. This limits the application of the considered methods to the detection of relatively shallow-lying defects which linear dimensions exceed the depth of their occurrence.

These methods can be used for thick-walled large-sized areas and constructions (various road and airfield coatings, heat-protective coatings made of foam and porous materials, walls of buildings and constructions, supporting bridge constructions, glued products). Of considerable interest for non-destructive testing of thick-walled large-sized constructions and structures made of composite materials are seismoacoustic methods that are widely used in seismic exploration when searching for minerals and studying rocks. The basis of these methods is the hodograph method of the velocity of elastic waves [7, 8, 11], which was most actively developed in the 60-70s of the last century.
The first mention of this method for non-destructive testing of thick-walled large-sized constructions and structures made of composite materials is given in [16]. There are no other publications on this method in relation to non-destructive testing of these objects.

It should be noted that the propagation velocities of longitudinal and transverse elastic waves are the most important parameters for non-destructive testing of physomechanical characteristics (elastic and strength) of medium materials [15].

**Methodology.** For a two-layer medium with a parallel interface, in which the propagation velocity of elastic waves in the first upper layer is much lower than the wave velocity in the second layer (Fig. 1), the hodograph equation takes the following form:

\[ v_2^2 (t - t_1)^2 = X^2 + 4\delta_2^2, \]  

(1)

where \( t_1 \) – the propagation time of the elastic wave in the first layer of the medium vertically.

In this medium, in addition to reflected waves, refracted waves can also be observed, while the maximum refraction effect is observed only if the layer thicknesses are equal. Hodograph equation for refracted waves \( w_{121} \) will be as follows:

\[ t = \frac{1}{v}(2\delta_1 \cos \alpha_0 + X \sin \alpha_0), \]  

(2)

where \( \alpha_0 \) – ultimate angle of refraction,

\[ \alpha_0 = \arcsin \frac{v_{w_1}}{v_{w_2}}. \]

When constructing the hodograph of refracted waves at a distance \( X_1 \) from the origin (Fig. 2), a kink of the straight line will be observed:

\[ X_1 = 2\delta_1 \sqrt{\frac{v_{w_1} + v_{w_2}}{v_{w_2} - v_{w_1}}}. \]  

(3)

From formula (3) it can be seen that the smaller the difference in the values of the velocities \( v_{w_1} \) and \( v_{w_2} \) is, the farther from the origin is the break point of fracture of the hodograph of refracted waves. In this case, the first part of the line before the hodograph fracture corresponds to the propagation of a direct longitudinal wave in the first layer, the second – in the second layer.

It can be seen from Fig. 2 and formulas (1) and (2) that the hodographs of reflected and refracted waves differ significantly from each other. Neglecting the interface, it can be shown that the shape of the hodograph of the reflected waves will not qualitatively change (see Fig. 1). So, the hodograph equation for a two-layer medium has the following form: 

**Fig. 1.** Scheme for constructing a hodograph of reflected waves in a two-layer medium

Em – emitter; Rec – receiver; \( v_1 \) – velocity of elastic waves in the first layer; \( v_2 \) – velocity of elastic waves in the second layer.

**Fig. 2.** Scheme for constructing a hodograph of refracted waves in a two-layer medium

Rec1, Rec2 – position of the refracted waves receivers; \( v_{w_1}, v_{w_2} \) – the speed of refracted waves in the first and second layer, respectively.
\[ t = t_0 + \frac{X^2}{4\delta(v_1 + v_2)}, \]  

(4) 

where \( \delta = \delta_1 = \delta_2 \); 
\[ v_{av} = \frac{\sum_{i=1}^{n} \delta_i}{\sum_{i=1}^{n} t_i} = v_{av} = \frac{\sum_{i=1}^{n} \delta_i}{\sum_{i=1}^{n} \delta_i}. \]

The approximation error is determined by the formula
\[ \Delta t = t' - t = \frac{1}{8} \frac{X^2}{4\delta^2} t_0 \left( \frac{1-n}{1+n} \right)^2, \]

where \( n = \frac{v_1}{v_2} \); \( t_0 \) – time at the emitter point.

Usually, the value of the approximation error \( \Delta t \) is taken to be equal to the error of measuring the time of the instrument. Associating the approximation error with the ratio of the distance between the emitter and the receiver to the thickness of the product, we obtain
\[ \frac{X}{2\delta} \leq \frac{2(1+n)}{1-n} \sqrt{\frac{2\Delta t}{t_0}}. \]

(5) 

Thus, formula (5) expresses the homogenization region of a two-layer medium into a homogeneous one, at which the hodograph of the reflected waves will not qualitatively differ.

For a multilayer isotropic medium with a parallel interface, the equations of the longitudinal linear hodograph of reflected waves are as follows:

\[ X = 2 \sum_{i=1}^{n} \frac{\delta_i v_i \sin \alpha_i}{\sqrt{v_i^2 - v_i^2 \sin^2 \alpha}}; \]

(6) 

\[ t = 2 \sum_{i=1}^{n} \frac{\delta_i v_i}{v_i^2 - v_i^2 \sin \alpha}, \]

(7) 

where \( \alpha_i \) – the angle between the direction of wave propagation (beam) and the normal in the first upper layer.

An analysis of expressions (6) and (7) shows that in order to obtain the equations of the linear hodograph of reflected waves in explicit form, it is necessary to expand expression (7) in a row, after which we obtain:

\[ t = t_0 + \frac{1}{2} \frac{X^2}{\sum_{i=1}^{n} 2\delta_i v_i} - \frac{1}{8} \frac{\sum_{i=1}^{n} 2\delta_i v_i^3}{\left( \sum_{i=1}^{n} 2\delta_i v_i \right)^4} X^4. \]

It should be noted that with \( \frac{X}{n\delta} < 0.5 \) the value of the third term of the equation can be neglected and limited to two terms of the expansion.

Of considerable interest is also the equation of the hodograph of refracted waves for a multilayer medium:

\[ t_n = t_{0n} + \frac{X}{v_n}, \]

where \( t_{0n} = 2 \sum_{i=1}^{n-1} \frac{\delta_i}{v_i} \cos \alpha_{i,n}; \alpha_{i,n} = \arcsin \frac{v_i}{v_n}. \)
In the process of propagation of elastic waves, especially longitudinal waves in a single-layer and multi-layer media, except for the once reflected, multiply reflected, refracted and mixed waves can also be observed. Consider the most characteristic features of the propagation of multiple waves, the main types of which are shown in Fig. 3.

In the general case of a homogeneous medium with parallel media interfaces, the equation of the hodograph of multiple reflected waves \( R_{101}, R_{10101}, \ldots, R_{1010101} \) has the following form (Fig. 3, a):  
\[
\nu^2 t^2 = X^2 + 4n^2\delta^2 ,
\]
where \( n \) – number of reflections.

In order to correctly distinguish between normal (once) reflected waves from multiple-reflected waves, which also have a hodograph of a hyperbolic shape (8), it is necessary to know some relationships between these waves. So, the abscissas of the minimum of the normal hodograph \( X_{01} \) and multiple hodographs \( X_{kn} \) are interconnected by the ratio \( X_{0n} = nx_{01} \), and the ratio of the propagation time at the point of the emitter for normal \( t_{01} \) and multiple \( t_{kn} \) waves has the form \( t_{0n} = nt_{01} \).

It should be noted that the main conditions for distinguishing normal waves from multiple-reflected waves are that the hodograph of multiple waves located at the same times as the hodograph of normal reflections from the upper interfaces are more abrupt. An effective means of recognizing normal waves is also an analysis of the amplitudes of normal and multiple waves at known coefficients of reflections from the interfaces.

The ratio of the amplitudes of multiple \( A_n \) and normal \( A_1 \) reflections for a medium with one intermediate interface can be written as follows:
\[
A_n/A_1 = (k_1k_2)^{n-1},
\]
where \( k_1 \) and \( k_2 \) – reflection coefficients, respectively, from the upper reflecting media interface and from the intermediate interface; \( n \) – multiplicity indicator (number of reflections).

The dependence (9) most clearly shows the ratio of amplitudes of multiple and normal waves if we compare the graphs for different reflection coefficients (Fig. 4). So, with \( k_1 = 0.2 \) and \( k_2 = 1 \) triple reflection should have an amplitude 25 times lower than a single reflection, and at \( k_1 = 0.2 \) and \( k_2 = 0.5 \) already 100 times smaller.

Thus, to highlight normal waves, you can use the following methods:

- tracking reflections by moving the receiver relative to the emitter;
- assessment of the ratio of the times of arrival of waves and the shapes of the in-phase axes;
• comparison of the abscissa of the minimum of normal and multiple reflected waves;
• selection of optimal $X/\delta$ values when tracking reflected waves in a layered and homogeneous medium;
• analysis of the ratio of amplitudes of normal and multiple reflections.

In addition to simple multiple reflections in a homogeneous medium, in the case of a multilayer medium with parallel interfaces consisting of two or more layers, complex types of multiple reflected waves and mixed waves (reflected-refracted and refracted-reflected) can arise.

For a two-layer medium with wave $R_{102}$ (see Fig. 3, b), using the previously considered principle, i.e., replacing the complex wave path with a simpler one and averaging the velocity values for each of the interfaces $v_{av1}$ and $v_{av2}$, we obtain the following expressions for determining the distance between the emitter $Em$ and the receiver $Rec$ and the wave propagation time:

$$X = 2(\delta_1 + \delta_2) \tan \alpha;$$

$$t = 2\left(\frac{\delta_1 + \delta_2}{v_{av1} + v_{av2}}\right) \sec \alpha.$$

where $\alpha$ – angle of wave reflection at the interface.

Excluding $\alpha$, we obtain

$$v_{av}t = 2(k_1 \delta_1 + \delta_2) \sqrt{1 + \frac{x^2}{4(\delta_1 + \delta_2)^2}},$$

(10)

where $k_i = v_{av2} / v_{av1}$.

Expression (10) is a hyperbole with a minimum at the origin. In this case, the time at the point of radiation for normal and multiple reflections from each medium interface is related by the dependence

$$t_0^{(2)} = t_0^{(1)} + t_{02}^{(1)},$$

where (1) and (2) – medium interfaces.

For a two-layer medium with wave $R_{212}$, the equation of the hodograph of the reflected waves has the following form:

$$v_{av2}t = 2(\delta_2 + k_1 \delta_1) \sqrt{1 + \frac{x^2}{4(\delta_2 + \delta_1)^2}},$$

time at the point of radiation for multiple and normal waves is determined by the dependence

$$t_0^{(2)} = t_0^{(1)} - t_{01}^{(1)}.$$

Based on the principle of reciprocity, the hodograph of the wave $R_m$ will coincide with the hodograph of the wave $R_{102}$.

The hodograph equation for a wave has the following form:

$$v_{av2}t = 2(2k \delta_1 + \delta_2) \sqrt{1 + \frac{x^2}{4(2 \delta_1 + \delta_2)^2}},$$

the relationship between the times at the point of emission

$$t_0^{(2)} = 2t_0^{(1)} + t_{02}^{(1)}.$$

In addition to the considered types of multiple waves, in a controlled medium can be observed waves of a mixed type – reflected-refracted ($R_{1212} > R_{3231323}$ and others). For this type of wave, we
consider a medium in which there is an interface that is simultaneously reflective and refractive (see Fig. 3, c). For such a medium, the equation of the hodograph of the reflected-refracted wave will have the following form:

\[ t = \frac{2\delta(N+1)}{v_1} \cos \alpha X + \frac{X}{v_2}, \]

where \( \delta = \delta_1 + \delta_2 + \delta_3; N \) — number of media interfaces.

Analysis of the hodograph equation of the reflected-refracted wave shows that on the time axis (at the point of radiation of Em) the hodograph of the refracted and reflected wave will cut off the following value:

\[ t_{\text{refr}} = \frac{2\delta \cos \alpha}{v_1}; t_{\text{refl}} = \frac{2\delta}{v_1}, \]

where \( \alpha = \arcsin \frac{v_1}{v_2} \); \( v_1, v_2 \) — velocity in the upper and underlying layer.

Thus, the smaller the difference between \( v_1 \) and \( v_2 \) the greater the difference between \( t_{\text{refr}} \) and \( t_{\text{refl}} \). Under real conditions, there may be a case where the refracting and reflecting boundaries do not coincide. This may cause wave interference. For this case, the condition for the intersection of the hodographs of the reflected and refracted waves can be written in the following form:

\[ \frac{2\delta_{\text{refr}} \cos \alpha}{v_{\text{av.refr}}} + \frac{X}{v_2} = \frac{1}{v_{\text{av.refl}}} \sqrt{X^2 + 4\delta_{\text{refl}}^2}, \]

where \( \delta_{\text{refr}} \) and \( \delta_{\text{refl}} \) — thickness of the layer of the medium in which the wave is refracted or reflected.

Having made some conversions we obtain

\[ 2b \cos \alpha + a \sin \alpha = c\sqrt{a^2 + 4}, \]  

where \( a = \frac{X}{\delta_{\text{av.refr}}}; b = \frac{\delta_{\text{av}}}{\delta_{\text{av.refl}}}; c = \frac{v_{\text{av.refr}}}{v_{\text{av.refl}}}. \]

Solving equation (11) with respect to \( a \), we obtain:

\[ a = \frac{2c}{c^2 - \sin^2 \alpha} \left( \frac{b \sin \alpha \cos \alpha \pm \sqrt{b^2 \cos^2 \alpha + \sin^2 \alpha - c^2}}{c} \right). \]

So, for the intersection of hodographs, reflected and refracted waves, to take place, the condition must be met:

\[ a \geq 2b \tan \alpha. \]

When analyzing the properties of direct and reflected waves, it was noted that for a homogeneous medium, the hodograph of a direct wave is parallel to the asymptotes of the hodograph of the reflected wave. However, in real conditions, the intersection of the hodograph of the direct and reflected waves can be observed. In this case, the equation for determining the distance \( X_1 \) from the emitter, at which the intersection will occur, will have the form:

\[ X_1 = \frac{2\delta}{\sqrt{v_{\text{av}}^2/v_0^2 - 1}}. \]

The main task of applying the low-frequency ultrasonic method is to determine the acoustic parameters of the propagation of elastic waves (velocities, amplitudes, spectra). The solution to the problem will clarify the determination of the coordinates of acoustic emission sources (AE) or the correct determination of the resource based on the registration of AE signals [1-4]. Consider the main methods for determining the velocities of elastic waves, based on the equation of the hodograph of the reflected waves. Replacing the coordinate values in it with quadratic coordinates \( u = (X - X_0)^2, w = t^2 \), we get a new hodograph equation

\[ u - v_1^2w + 4\delta^2 = 0. \]
This equation is an expression of a straight line, the angular coefficient of which determines the square of the velocity of propagation of longitudinal waves in the first layer, and the segment cut off by this straight line along the $t$ axis is $2\delta$ value. Moreover, in the case of parallelism of the external and reflective surfaces $X_0 = 0$.

The hodograph of reflected-refracted waves presents a considerable practical interest in determining the velocities of elastic waves in layers. In accordance with formula (8), it seems possible to determine the velocity of longitudinal or shear waves in almost every layer of a multilayer isotropic medium (Fig.5).

To calculate the elastic wave velocities in the layers, the following formulas can be used:

- for two-layer medium ($w_{121}$)
  $$v_1 = \frac{2\delta_1 \cos \alpha_{1,2}}{t_{02}}; \quad v_2 = \frac{X}{t_2 - t_{02}};$$

- for three-layer medium ($w_{12321}$)
  $$v_3 = \frac{2\delta_2 \cos \alpha_{2,3}}{t_{03} - \frac{2\delta_1}{v_1} \sin \alpha_{1,3}};$$

- for four-layer medium
  $$v_4 = \frac{2\delta_3 \cos \alpha_{3,4}}{t_{04} - \left(\frac{2\delta_1}{v_1} \cos \alpha_{1,4} + \frac{2\delta_2}{v_2} \cos \alpha_{2,4}\right)};$$

- for $n$-layer medium ($w_{12... n ... 21}$)
  $$v_n = \frac{2\delta_{n-1} \cos \alpha_{n-1,n}}{t_{0n} - 2\sum_{i=1}^{n-1} \frac{\delta_i}{v_i} \cos \alpha_{i,n}}.$$

The numbers at the angle of reflection $\alpha$ indicate the interface between the media.

It should be noted that to determine the velocity in the layer $n$, it is first necessary to determine the velocities in all overlying layers, starting from the first.

A transversely isotropic medium is characterized by isotropy in the plane and in the direction at an angle to the plane of the product. The degree of anisotropy of the speed of elastic waves will be determined by the ratio of the speed of elastic waves in the vertical and horizontal directions. Since the propagation time, and, consequently, the velocity is a function of the angle between the direction of propagation and the plane of elastic symmetry of the medium, the equation of the linear hodograph of reflected waves for a transversely isotropic medium takes the following form:

$$t(\alpha) = \frac{\sqrt{\sin^2 \alpha + \alpha_n^2 \cos^2 \alpha} (X^2 + 4\delta_2)}{\alpha_n v_{n90}}$$

where $\alpha_n = v_{n90}$, $v_{n0}$ and $v_{n90}$ — longitudinal wave velocities in the plane of the layers and perpendicular to the layers.
In the case of an isotropic medium, the equations of the hodograph of the longitudinal and transverse reflected waves have the same form, they are replaced only depending on the type of waves, the velocity values (longitudinal or transverse). In the case of a transversely isotropic medium, the hodographs of longitudinal reflected and transverse reflected waves differ significantly, since the velocity of longitudinal waves with a change in the angle of reflection from 0 to 90° decreases, and the velocity of the shear waves with a change in the angle from 0 to 45° increases from 45° to 90° decreases. Thus, the velocity of longitudinal waves has two extreme values – at 0 and 90°, and the velocity of transverse (shear) waves has three extreme values at 0°, 45° and 90°. Wherein \( v_0 = v_{n90} \), and the characteristic, the degree of velocity anisotropy is the expression \( a_n = \frac{v_0}{v_{45}} \). Then the equation of the hodograph of the transverse (shear) reflected waves takes the form

\[
\tau(\alpha) = \frac{\sqrt{\sin^2 \alpha + a_n^2 \cos^2 \alpha} (X_2^2 + 4\delta^2)}{\alpha c v_{c45}}.
\]

In the practice of non-destructive testing the control of two-layer structures, which can be used as a combination of a transversely isotropic layer with an isotropic one, is of considerable interest. In this case, the emitter \( Em \) and the receiver \( Rec \) are optimally positioned on the side of the layer having a lower speed. The hodograph and the propagation scheme of elastic waves in such a medium are shown in Fig.6.

A fracture of the hodograph line occurs when the propagation time of longitudinal waves in the first layer along the Em-Rec profile is equal to the propagation time of longitudinal waves in the layer along the line Em-A-B-Rec, i.e.

\[
\frac{l}{v_{n90}} = \frac{2(\sin^2 \alpha + a_n \cos^2 \alpha) \sqrt{\delta_1^2 + X_1^2}}{v_{n90} a_n^2} + \frac{l - 2X_1}{v_{n2}}.
\]

Proceeding from the Fermat principle based on minimizing the propagation time, the hodograph equation for refracted longitudinal waves in a transversely isotropic medium will have the following form:

\[
\tau = \frac{2(\sin^2 \alpha + a_n \cos^2 \alpha) \sqrt{\delta_1^2 + X_1^2}}{a_n^2 v_{n90}^2} + \frac{l - 2X_1}{v_{n2}},
\]

where \( X_1 = \text{Em} A \sin \alpha \).

As \( \sin \alpha = \frac{v_{n90}}{v_{n2}} \), then \( X_1 = \frac{v_{n90}}{v_{n2}} \sqrt{\delta^2 + X_1^2} \) or \( X_1 = \frac{v_{n90} \delta}{\sqrt{v_{n2}^2 - v_{n90}^2}} \).

The distance at which a hodograph line fracture should be expected is determined by analogy with expression (3) by the formula

\[
l_0 = 2\delta \sqrt{\frac{v_{n2} + v_{n90}}{v_{n2} - v_{n90}}},
\]

Velocity values \( v_{n90} \) and \( v_{n2} \) can be determined by the tangents of tilt angels \( \text{tg} \chi_1 \) и \( \text{tg} \chi_2 \).
Conclusion. The proposed hodograph method allows one to determine the propagation velocity of elastic waves in each layer of a multilayer medium, which provides non-destructive control of the physicomechanical characteristics (elastic and strength) of materials in each layer of a multilayer medium.

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