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CALCULATION OF ELASTOVISCOPLASTIC DISPLACEMENT OF WELL WALLS IN TRANSVERSAL AND ISOTROPIC ROCKS

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The relevance of the work is justified by the need to improve the technical and economic indicators of well construction based on forecasting and preventing drilling tools sticking due to the narrowing of an open well bore in the intervals of transversely isotropic rocks.

A mathematical model of elastic-viscous-plastic displacement of the walls of inclined and horizontal wells has been developed during the narrowing of the open borehole due to rock creep in the intervals of transversely isotropic rocks. In the program developed based on this mathematical model, the calculation of the elastic-viscous-plastic displacement of the walls of an obliquely directed and horizontal well in the reservoir of argillite from the Western Siberia deposit was carried out. As a result of the calculation, it was established that after opening the rock with bits, the cross-section of the open borehole due to the rock creep eventually takes the form of an ellipse, the small axis of which is in the plane of the upper wall of the well and decreases with time.

Key words: bit sticking; instability of the walls of the wells; open hole narrowing; transversely isotropic rock; elastic-viscoplastic movement of the borehole walls

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Introduction. The reasons for the decrease in technical and economic indicators of drilling oil and gas wells are complications, among which the most frequent and significant ones are caused by the sticking of the drilling tool. Among the causes of sticking, we can distinguish violation of the stability of an open well bore (narrowing of the trunk, cavern formation, cavings and walls collapsing) [2, 4, 9, 11-15].

Formulation of the problem. One of the main causes of drilling tool sticking in inclined and horizontal wells is the narrowing of the open hole in the intervals of viscous-plastic rocks [2, 4]. Earlier several authors [3-6, 9] considered modeling of the process of elastic-viscous-plastic displacement of the walls in the inclined and horizontal wells due to creep of rocks composing the walls of the well. However, these papers did not consider the effect of the anisotropy of the elastic properties of the rock. In this article, we consider the problem of modeling the elastic-viscous-plastic displacement of the wall of inclined and horizontal wells in a transversely isotropic compressible rock.

Research methods. We used methods of the theory of elasticity, the theory of linear visco-elasticity, rock mechanics, and rock creep theory. The problem was solved by the methods of numerical integration and variable modules under the assumption of planar deformation of compressible elasto-viscoplastic rock. The rock creep is modeled by linear hereditary media, the parameters of which are determined by the Abel core. The solution of the problem was obtained analytically based on the field data of geophysical studies of the wells and the published rock creep results.

Discussion. The computational model of the problem is shown in Fig.1. The rock is transversally isotropic, and the *z*-axis is located along the normal to the isotropy plane. A drilling fluid is pumped into the open wellbore, creating hydrostatic pressure on the well walls.

While drilling, the bit forms a round well with radius R_n, but as the bit moves away from the fixed cross-section of the well, the wellbore deforms, as shown in section *I-I* (Fig. 1) [2, 4]. Due to the rock creep, it moves in time in the radial direction by the value of $u(t)$, which ultimately leads to the displacement of the borehole walls in time. In the direction *AD*, a positive movement in time of point *A* and, accordingly, the sidewall of the well occurs (the well radius increases), and in the direction *BE*, a negative movement of point *B* and, correspondingly, the upper wall of the well to the axis of the well [2, 4]. From this, it follows that the points *A* on the side wall and *B* on the upper wall are characteristic, the calculation of the displacement of which makes it possible to determine the large and small axes of the elliptical section of the well due to non-axisymmetric deformation [2, 4].

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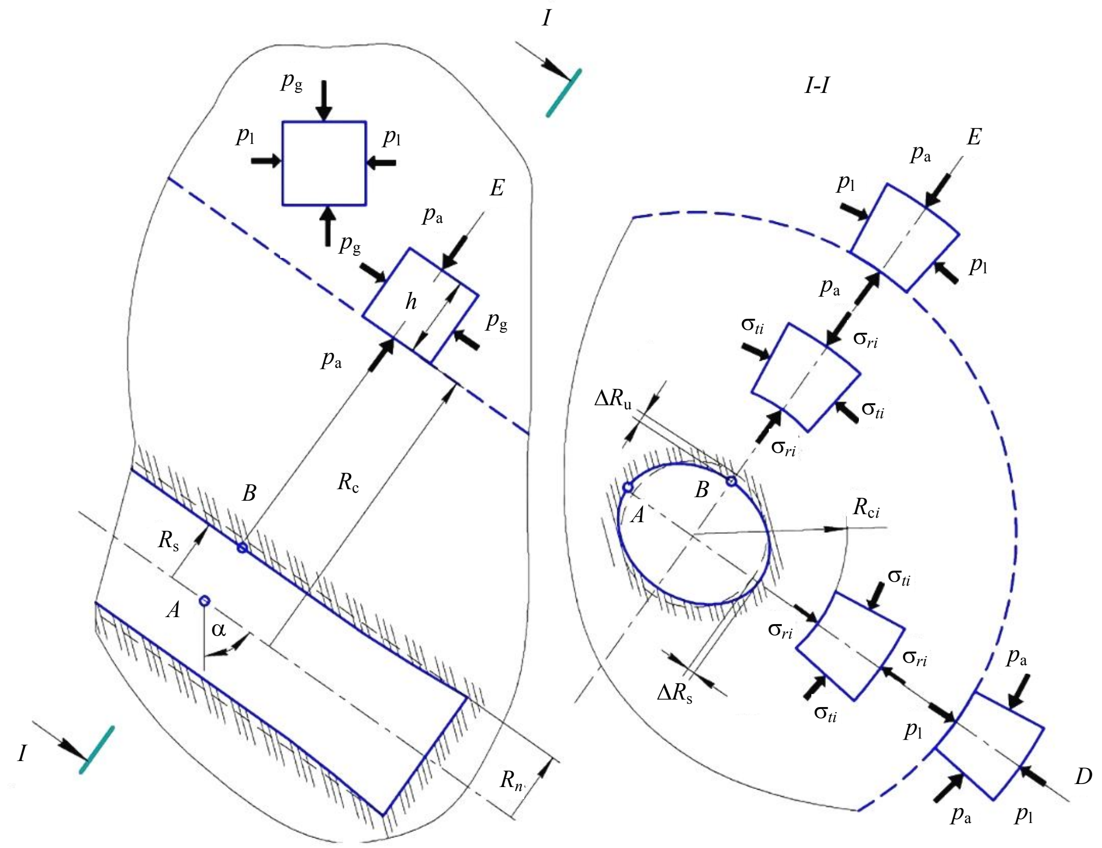

Fig.1. Calculation model of the problem

Point *A* – side wall of the well; point *B* – upper wall of the well; points *E* and *D* of the calculated contour at the near-wellbore zone; α – inclination angle of the well interval; p_g – geostatic rock pressure; p_l – lateral rock pressure; p_a – pressure on the *i*-th element of the rock mass due to the action of the rock pressure field; *h* – thickness of the *i*-th element of the rock mass; σ_{ri}, σ_{ti} – radial and tangential stress respectively *i*-th element of the rock mass; R_{ci} – the radius of the contour of the *i*-th element of the rock mass; R_n – nominal open hole radius (equal to half the bit diameter); R_s – sidewall radius; R_u – upper wall radius; R_c – impact contour radius in the near-wellbore zone; ΔR_s and ΔR_u – elastic-viscoplastic displacement, respectively, of the side and upper walls of the well [2, 4]

The numerical solution of the problem of elastic-viscous-plastic displacement of the wall of inclined and horizontal wells was obtained under the assumption of planar deformation of the rocks surrounding the well, i.e. the change in strain in the direction parallel to the axis of the well is zero. The problem is solved in a cylindrical coordinate system $R\theta Z$, in which the applicate Z is vertical and coincides with the axis of the well, and the R axis is directed in the radial direction and normal to the Z-axis [2, 4]. It is assumed that the isotropy plane $R\theta$ coincides with the analogous plane of the cylindrical coordinate system used to solve the problem.

Boundary conditions: 1) in the intact rock mass the axial stress equals geostatic pressure $(\sigma_z = -p_g = \text{const})$, radial and tangential stress is $\sigma_r = \sigma_t = -p_l = \text{const}$; 2) after opening the rock with bits at the same depth on the borehole wall ($r = R_s$) the radial stress equals the hydrostatic pressure of the drilling flushing fluid p_s $(\sigma_r = -p_l)$, at a distance of the radius of impact contour ($r = R_c$), radial and tangential stresses are equal to the lateral rock pressure ($\sigma_r = \sigma_t = -p_l = \text{const}$),and the axial stress is equal to geostatic pressure $(\sigma_z = -p_g = \text{const})$, the radial displacement at the radius of the impact contour is zero ($u = 0$) [2, 4].

The radius of the impact contour R_c is taken from the condition that at about ten radii of the wellbore, the stress in the near-wellbore zone tends to strain in conditions of natural occurrence [2, 4].

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The rock creep is given by a linear hereditary elastic medium model [7, 10]. As a creep function, the Abel core was taken, which coefficients are determined from the results of tests of rock samples for creep [7].

At the first step, the solution of the radial elastic displacement is obtained based on the geometric and physical equations of the theory of linear elasticity [2, 4, 10]. The radial elastic displacement of the *i*-th element of the rock mass is determined by the formula [2, 4]

$$\Delta R_{y,i} = \varepsilon_r h = \left(\frac{1}{E}\left(\Delta\sigma_{r,i} - \mu\Delta\sigma_{\theta,i}\right) - \frac{\mu_1}{E_1}\Delta\sigma_{z,i} \right) h , \qquad (1)$$

where ε_r – radial deformation of the *i*-th element of rock mass; E – modulus of elasticity for directions in the isotropy plane $R\theta$; E_1 – modulus of elasticity for directions normal to the isotropy plane $R\theta$; μ and μ_1 – Poisson's coefficients for deformations in the isotropy plane under compression, respectively, in the isotropy plane $R\theta$ and in the directions normal to the isotropy plane $R\theta$; h – the thickness of the *i*-th element of the rock mass [2, 4].

The increment of the axial stress in the elastic rock is determined by the formula

$$\Delta\sigma_{z,i} = \frac{D(p_l - p_c)}{R_{0i}^4} - \frac{2(p_a - p_l)}{R_{0i}^2} , \qquad (2)$$

where $\Delta\sigma_{z,i}$ – the increment of the axial stress of the i-th element of the rock mass; $R_{0i} = R_i / R_n$; $D = (1+\mu)^2 / [16E(1-\mu)^2(1-2\mu)]$; $p_a = p_g \sin^2 a + p_l \cos^2 a$.

The increments of radial and tangential stresses of the i-th element of the rock mass are determined by the formulas of S.G.Lekhnitsky [4].

At the second stage, the problem of determining the viscoplastic displacement of the borehole wall is solved. The solution of the problem is obtained by the method of variable modules developed by B.Z. Amusin and A.M. Linkov, according to which the elastic constants in equation (1) are replaced by algebraic expressions containing the creep function (temporary functions of the corresponding parameters), provided that the boundary conditions and geostatic pressure do not depend on time [1].

When calculating the viscoplastic deformation of the rock in accordance with the method of variable modules, the coefficient D in the formula (2) is replaced by the time function *D(t)*:

$$D(t) = \frac{(1+\mu_t)^2}{16E_t(1-\mu_t)^2(1-2\mu_t)} ,$$

where E_t and μ_t – time function, respectively, of the elastic modulus and Poisson’s ratio in the isotropy plane [7, 10].

Viscoplastic radial displacement of the i-th element of the rock mass in accordance with the method of variable modules is determined by the formula:

$$\Delta R_{vp,i} = \varepsilon_r(t) h = \left(\frac{1}{E_t}\left(\Delta\sigma_{r,i} - \mu_t\sigma_{\theta,i}\right) - \frac{\mu_{1t}}{E_{1t}}\Delta\sigma_{z,i} \right) h , \qquad (3)$$

where $\Delta R_{vp,i}$ – viscoplastic component of the movement of the i-th element of the rock mass; $\varepsilon_r(t)$ – radial viscoplastic deformation of the i-th element of the rock mass; μ_{1t} and E_{1t} – time function of Poisson’s ratio and elastic modulus, respectively, in directions normal to the isotropy plane [7, 10].

Elastoviscoplastic displacement of the i-th element of the rock is determined by the sum of the elastic component and the component due to viscoplastic displacement. The elastic-viscoplastic displacement of the borehole wall is determined by the total elastic-viscoplastic displacement of all elements:

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$$R_c(t) = R_{e.c} + R_{c.vp}(t) = \sum \Delta R_{e,i} + \sum \Delta R_{vp,i}(t), \quad (4)$$

where $R_{e.c}$ – elastic component of the displacement of the borehole wall; $R_{e.vp}(t)$ – viscoplastic component of the displacement of the borehole wall.

As a result of the substitution of equations (1) and (3) into equation (4) and subsequent mathematical transformations, we obtain the equation for the elastic-viscous-plastic movement of the walls of inclined and horizontal wells:

$$R_c(t) = \sum\left(\left(\Delta\sigma_{r,i} - \mu\Delta\sigma_{\theta,i}\right)\frac{E+E_t}{EE_t} - \Delta\sigma_{z,i}\left(\frac{\mu_1 E_{1t} + E_1\mu_{1t}}{E_1 E_{1t}}\right)\right)h.$$

Results. Based on the solution obtained, calculations of the elastic-viscous-plastic displacement of the wall of inclined and horizontal wells are made on the example of a field of Western Siberia. The following data were taken as the initial data: rock - argillite; nominal diameter of the borehole (equal to the diameter of the bit) 215.9 mm; geostatic pressure is 62 MPa, hydrostatic pressure of drilling fluid is 27 MPa; the elastic moduli of the rock are $E = 4900$ MPa, $E_1 = 7600$ MPa, the Poisson coefficients of the rock are $\mu = 0.22$ and $\mu_1 = 0.15$. The coefficients of the Abel core of the rock, respectively, $\delta = 0.08$ s^{-1}, $\alpha = 0.71$ [7, 8].

On the basis of the initial data, the dependence of the elastic-viscoplastic displacement of the wall of a horizontal well after opening the rock with bits at the inclination angle of 90° is obtained (Fig.2). At the initial moment of time, the displacement of the borehole wall occurs in accordance with the equations of linear elasticity and is not equal to zero. Next, there is a viscoplastic movement of the borehole wall due to rock creep in the near-wellbore zone. As can be seen from Fig.2, the elastic-viscous-plastic movement of the upper wall of the well during the day after opening the rock increases more than 3 times, while the same movement of the side wall of the well during the same period remains almost unchanged. One day after the rock is opened with a chisel, the elastic-viscoplastic movement of the upper wall of the well is more than 6 times higher than the elastic-viscoplastic movement of the side wall of the well. Accordingly, the cross-section of an open hole of inclined well eventually takes the form of an ellipse, the major axis of which lies in the plane of the side walls of the well, and the minor axis in the plane of the upper wall of the well.

The dependence of the elastic-viscoplastic displacement of the walls of inclined and horizontal wells on the inclination angle of the interval was determined according to the results of the calculation a day after the rock was opened with a chisel (Fig.3). It is established that with an increasing inclination angle, the elastic-viscoplastic displacement of the upper and side walls increases. Thus,

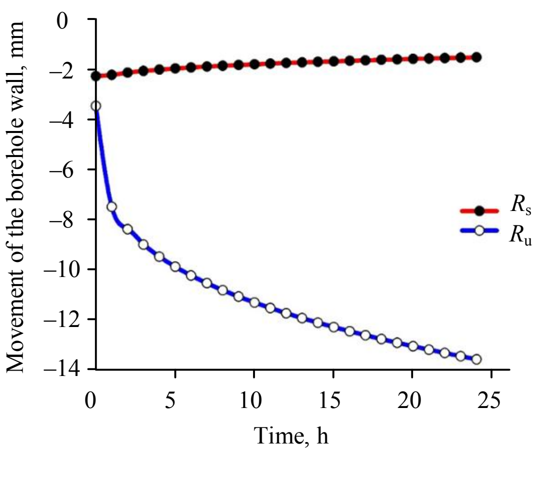

Fig.2. Elastic-viscoplastic displacement in time of the wall of inclined well

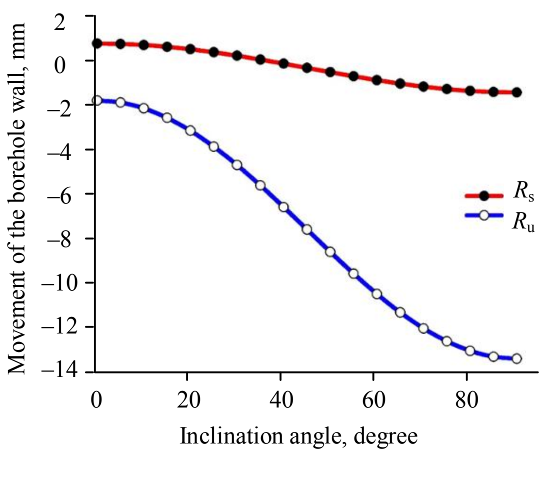

Fig.3. Dependence of elastic-viscoplastic displacement of the borehole wall from the inclination angle of the interval

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the elastic-viscoplastic displacement of the upper wall of a horizontal well (inclination angle 90°) is more than 6 times greater than the elastic-viscoplastic displacement of the upper wall of a vertical well (inclination angle is zero).

Conclusions

1. The numerical analytical solution is obtained for the problem of the elastoplastic displacement of the walls of inclined and horizontal wells in the reservoir of a compressible elastoviscoplastic transversely isotropic rock.

2. As a result of the calculation based on the obtained mathematical model, we determined the dependences of the elastic-viscous-plastic displacement of the walls of inclined and horizontal wells on the time after the rock was opened with a rock-breaking tool and on the inclination angle of the interval in the argillite formation of the Western Siberia.

3. It is shown that over time and with increasing inclination angle, the value of the elastic-viscous-plastic movement of the upper wall of the well is a multiple of the value of the elastic-viscoplastic movement of the side wall of the well, resulting in the cross-section of an open borehole acquiring the shape of an ellipse with a small axis.

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