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Assessment of internal pressure effect, causing additional bending of the pipeline

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Article justifies accounting for internal pressure effect in the pipeline, causing additional bending of the pipeline. According to some scientists, there is an erroneously used concept of the equivalent longitudinal axial force (ELAF) S_x, which depends on working pressure, temperature stresses, and joint deformations of pipelines with various types of soils. However, authors of the article use ELAF S_x concept at construction of mathematical model of stress-strain state (SSS) for complex section of the trunk pipeline, and also reveal it when analyzing the results of calculating the durability and stability of the pipeline. Analysis of SSS for calculated section of the pipeline was carried out for two statements of the problem for different values of operation parameters. In the first statement, effect of internal pressure causing bending of the pipeline is taken into account, and in the second it is neglected. It is shown that due to effect of ELAF S_x at p_0 = 9.0 MPa, Δt = 29 °C extreme value of bend increases by 54 %, extreme values of bending stresses from span bending moment increase by 74 %, and extreme value of bending stresses from support bending moment double with regard to corresponding SSS characteristics of the pipeline. In case of neglecting the internal pressure effect causing additional bending of the pipeline (second statement of the problem), error in calculating the extreme value of bend is 35 %, extreme value of bending stresses from span bending moments is 44 %, and extreme value of bending stresses from support bending moments is 95 %.

Key words: oil pipeline, pipeline, soil, bend, force, moments, stress, strain, pressure

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Introduction. This article is devoted to justification of considering the internal pressure, which causes additional bending of pipeline, and refutation of claims in works of some authors [2, 4, 5] about erroneous introduction of ELAF S_x concept, determined depending on internal working pressure, temperature stresses, and joint deformations of the pipeline with various types of soils in the works of I.P.Petrov and V.V.Spiridonov [11], A.B.Ainbinder and A.G.Kamershtein [1], as well as in regulatory document, which governs the construction and operation of trunk pipelines [16]. This justification will be given when constructing a mathematical model of SSS for a complex trunk section of the pipeline, and will also be disclosed when analyzing the results of durability calculation for the pipeline. Next, necessary information about statement of the problem and its solving method will be provided [12].

Statement of the problem. Profile of calculated section of the pipeline is figuratively divided into parts in which geometric and stiffness characteristics of the pipe are constant, loading conditions and soil conditions are identical or change smoothly and without fluctuations. Mathematical model of the pipeline is a one-dimensional rod system consisting of curvilinear and rectilinear rods of a tubular section and their interface nodes [8, 12]. Figure 1 schematically shows longitudinal axis

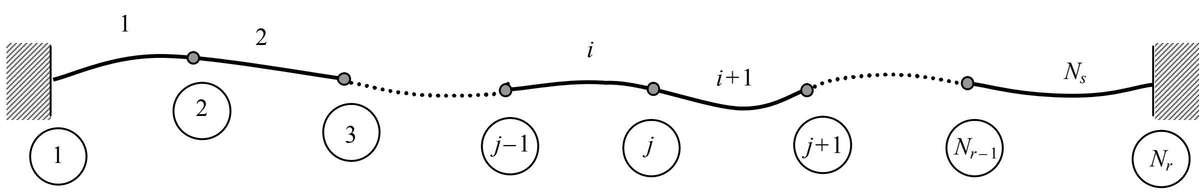

Fig.1. Calculation scheme of longitudinal axis of the rod system

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of the rod system, figuratively divided into rod and nodal elements, which are numbered as follows: i – number of rod element; N_s – their total amount ($1 \le I \le N_s$); j – number of nodal element; N_r – total amount of nodes ($1 \le j \le N_r$). If there are no branches in the pipeline route, then $N_r = N_s + 1$.

Figure 2 shows a rod element of length dx, which, after loading by external forces, transfers from lower to upper position. Directions of the axes for curvilinear coordinate system are also indicated here. Being deformed together with longitudinal axis of the rod element, system accompanies the rod when it is displaced.

Compression-tensile strain of longitudinal axis for the rod ε_x, rotation angle of its normal ω_x, bending strain χ are related to its longitudinal displacement u and bending w using kinematic non-linear differential relations [8, 12]:

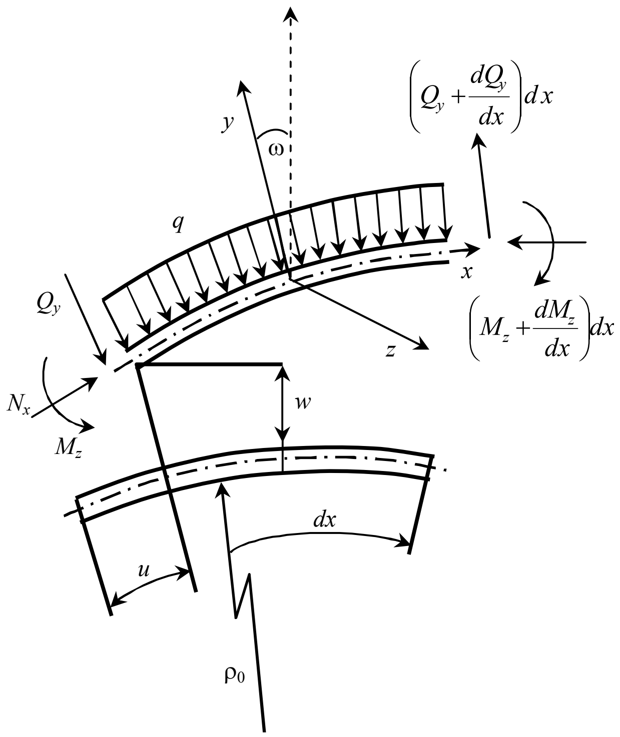

Fig.2. Calculation scheme of the rod element

$$\varepsilon_x = \frac{du}{dx} - \frac{w}{\rho_0} + \frac{1}{2}\omega_x^2; \quad \omega_x = -\frac{dw}{dx} + \frac{u}{\rho_0}; \quad \chi = \frac{d\omega_x}{dx}, \tag{1}$$

where ρ_0 – radius of curvature for longitudinal axis of the rod before its deformation (radius of curvature for the branch); x – axial longitudinal coordinate of the rod.

Derivation of kinematic (geometric) non-linear differential relations (1), performed by authors of the article, is presented in [12]. They introduce an accompanying coordinate system, rigidly connected with axial line of the rod, which, deforming with this axis, moves with it in space. Further, according to the theory of differential geometry and the theory of non-linear deformations of the rods, relationships between Serret – Frenet trihedra of longitudinal axes for the rod are established before and after its deformation, which allows obtaining main kinematic (geometric) non-linear differential relations connecting the tensile-compression, bending and torsion strains of longitudinal axis for the rod through coordinates of displacement vector u. If we assume that the rod is in a plane-deformed state, i.e. deformation of the rod occurs without torsion, then non-linear differential relations are simplified and can be represented in the form of a formula (1).

In book [1], when calculating SSS of a pipeline with curved inserts, the latter are replaced by straight pipes connected at an angle, i.e. position of longitudinal axis for curved insert in space is defined by a broken line. Geometric relations (1), in contrast to analogous relations of [1] for rectilinear rods, were obtained taking into account the initial curvature of longitudinal axis for the rod. Therefore, in calculations of SSS for pipelines with curved inserts (branches) according to our developed mathematical model, there is no need to replace curved inserts with broken ones, which is carried out in methods using the theory of calculating SSS for pipelines in [1]. Calculations performed in [12] showed that such a replacement can lead to incorrect results, since model of broken lines instead of a curved insert cannot adequately describe SSS of a pipeline with curved inserts and loss of stability when it is compressed in longitudinal axial direction.

Equilibrium equations of the rod simulating SSS of a separate part of figurative partition for underground section of the pipeline are as follows [8, 12]:

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$$\frac{d\left(N_x+\omega_x Q_y\right)}{dx}-\frac{Q_y}{\rho_0}=r_x+q_x-q_y\omega_x;$$

$$\frac{dQ_y}{dx}+\left(\frac{1}{\rho_0}+\chi\right)\left(N_x - p_0F_{\text{il}}\right)=r_y+q_y; \qquad (2)$$

$$\frac{dM_z}{dx}-Q_y=0,$$

where N_x, Q_y – respectively, longitudinal axial force and cutting force directed along the tangent and normal to deformed longitudinal axis of the rod; M_z – axial bending moment, under effect of which the longitudinal axis of the rod is twisted around the binormal in plane of drawing; r_x, r_y – respectively, tangent and normal components of soil reaction to longitudinal movement and bend of the pipe; q_x, q_y – respectively, longitudinal and normal components of external distributed load, consisting of the weight of pipeline with liquid or gas, as well as pressure (weight) of soil backfill on the pipe; p_0 – internal working pressure; F_{il} – cross-sectional area of the pipe wall "in light".

Physical relationship between force factors and tensile-compression and bending strains is taken in the form of the following relations [8, 12]:

$$N_x=(E\,\varepsilon_x+\mu\,\sigma_{\text{an}}-\alpha\,\Delta t\,E)F_0;\; M_z=E\,J\chi, \qquad (3)$$

where E, μ – accordingly, a variable parameter of elasticity (Young's modulus) and a variable coefficient of transverse deformation of steel (Poisson's ratio), determined by the formulas of regulatory document [16] taking into account deformation of steel pipe in elastoplastic area; α – coefficient of linear expansion of pipe steel; σ_{an} – annular stresses from internal working pressure; Δt – temperature difference; F_0 – pipe wall cross-sectional area; J – moment of inertia for the pipe cross section.

When compiling differential equations of equilibrium in vector form, difference between total derivatives and partial derivatives with respect to time is taken into account, which leads to appearance of a longitudinal component in them from internal pressure acting perpendicular to the pipe wall [12, 15]. This method of compiling a differential equation in vector form was first used by V.A.Svetlitskii [15]. When describing the kinematic relations (1), it will be correct to design compiled equilibrium equations and boundary conditions in vector form on deformed longitudinal axis of the rod. Only in this case components of the force vector will have a clear physical meaning: they will represent cutting force and longitudinal forces. Then, in solving the problem by finite elements method, there is no need to find coordinates of interface nodes, since in accompanying coordinate system, rigidly connected with longitudinal axis of the rod system, coordinates of this line do not change when the latter is deformed.

Energy research methods used in stability problems of pipelines operated in difficult engineering and geological conditions cannot adequately describe their SSS, since the adopted form of buckling determines solution to the problem [1]. Authors of [8, 12] developed methods for calculating SSS and stability of underground pipelines with a complex outline of longitudinal axis for the pipe using the finite elements method taking into account joint deformation with soil and operating parameters. Since problem of SSS for an individual rod element is solved in a geometrically non-linear formulation and takes into account physical non-linear behavior of soil, there is no need for additional verification of longitudinal stability of the pipeline. In this article, equilibrium equations (2) are composed in a curved coordinate system, rigidly connected with unit vectors of longitudinal axis for deformable pipeline. In [8, 12], a complete description of numerical method for solving the problem was given, in which three non-linear differential relations (1) and three non-linear differential equations (2) are presented in the form of a normal system of six ordinary first-order non-linear differential equations. At each junction node, a system of three algebraic equilibrium equations is composed taking into account effect of concentrated forces and moments applied at these nodes, as well as limitations of longitudinal displacement, angle of rotation for the normal, and bending. This system of equations is supplemented by boundary conditions set at beginning and end of calculated section. Normal system of non-linear differential equations for finding the elements of stiffness matrix, components for the vector of common efforts and SSS of an individual rod element was solved by method of orthogonal sweep of academician S.K.Godunov [3]. When finding movements of nodal elements, displacement method developed by A.V.Aleksandrov for calculating shell structures is used, in which, unlike standard methods of finite differences and finite elements, by dividing the calculated structure into nodal and rod elements, resolving system of algebraic equations is compiled in junction nodes of these elements, taking into account design features of the pipeline. This allows reducing the main resolving system of algebraic equations by several orders of magnitude. Using this method V.I.Myachenkov developed a theory and a set of programs for calculating the durability and stability of structures, which are modeled by a composition of thin-walled elastic shells of various shapes and stiffening rings (frames) [9]. Authors of presented article adapted them to calculate SSS of vertical cylindrical tanks with corroded walls, reinforced by frames during repair work [8].

The following should be noted. If, when compiling the equations, we restrict ourselves to considering partial and not total time derivatives, then equilibrium equations will not contain longitudinal component of internal pressure. Unfortunately, many compilers and users of commercial software do not pay attention to this and use general differential equations, which are obtained without taking into account the difference between full derivatives and partial derivatives in time, which leads to neglect of internal pressure effect in durability and stability calculations that causes additional bending of the pipeline.

Necessity to take into account longitudinal component from internal pressure effect on pipe wall during its bending in equilibrium equations was proved in works of V.A.Svetlitskii [15], M.A.Ilgamov [6, 7], Tang [19], V.I.Feodosev [17]. They examined various cases of fixing the ends of the pipe, justified the possibility of losing its stability. For example, M.A.Ilgamov [6, 7] when compiling the equilibrium equation drew attention to the fact, which is as follows. When the pipe is bent, area on which internal pressure p_0 acts on convex side of the pipe surface is larger than on concave one. Thus, when the pipe bends, an excess surface is formed. Resultant of pressure on this excess surface will be equal to $p_0F_{\text{il}}w''$, under the influence of this resultant, pipe undergoes additional bending. If there is no pipe bend ($w'' = 0$), then there will be no excess area, so internal pressure in the pipe will not cause its additional bending.

Fact of introducing ELAF S_x concept is quite justified, determined depending on internal working pressure and temperature stresses, in works of I.P.Petrov and V.V.Spiridonov [11], A.B.Ainbinder and A.G.Kamershtein [1], as well as in regulatory document governing the construction and operation of trunk pipelines [16]. This force is not quite correctly called the equivalent longitudinal axial force, but direction of its effect and magnitude in differential equations are described correctly. In [1, 8, 12], an equation is presented that describes SSS of a pipeline in a single-span beam crossing without compensation for longitudinal displacements, which coincides with equation of longitudinal-transverse bending of the beam under effect of vertical load and longitudinal compressive force applied at its ends [18]. In [8, 12, 18], detailed study of solution for this equation was carried out, as well as equations of longitudinal-transverse bending for a tensile rod simulating SSS of a pipeline with a compensator.

According to some researchers [2, 4, 5], equation of longitudinal and transverse bending of the pipeline, magnitude of longitudinal force in [1, 8, 11, 12], as well as formulas for checking longitudinal stability of trunk pipelines contained in regulatory document [16], are composed with significant errors. If these conclusions are adopted, computer programs for calculating the durability and stability of pipelines developed by staff of VNIIST, Giprospetsgaz, VNIIGAZ and YUZHNIIGIPROGAZ teams over the past 50 years, as well as results of study of other scientists that take into account the effect of internal pressure on bending of the pipeline in accordance with requirements of regulatory document [16] articles, should be considered performed according to incorrect equations and erroneous formulas. Authors of [2, 4, 5], incorrectly taking into account longitudinal force arising from internal pressure effect, obtained differential equation of pipe bending, which is stretched by longitudinal force. They consider impossible the loss of stability for the pipeline with increasing internal pressure, whereas the pipe stretches like a stretched string. Same authors state that “value of ELAF S_x “penetrated” into more important formulas describing longitudinal-transverse bending of pipelines. Based on this formula, technological modes of pipelines' operation are determined. An incorrect initial formula leads to erroneous results, and, consequently, to unforeseen cases (incidents and emergencies) during repair of pipelines”.

Absence in [2, 4, 5] of analysis for solution of differential equation obtained by authors of these works led to wrong results and incorrect conclusions. They did not take into account well-known fact that follows from solution of this equation: an increase in internal pressure leads to a decrease in calculated characteristics of the pipeline SSS. Therefore, a conclusion is obtained that contradicts common sense: to ensure durability of the pipeline, it is necessary to increase internal pressure in the pipe. This example of a mistake in statement of the problem, which led to significant errors in calculation results and conclusions, is not the only case among researchers dealing with questions of durability and stability of the pipelines. In calculation of pipe stability in a single-span beam crossing without compensation for longitudinal displacements, when finding a longitudinal force, some of the researchers use the formula according to which an increase in the internal pressure in the pipe reduces calculated value of longitudinal force. In calculating SSS of the pipeline, they do not use differential equation of longitudinal-transverse bending of the rod during its compression, with which it is possible to find the value of critical compression force of the pipe, instead they use the solution of the pipe bending equation, which does not contain a longitudinal force when it is compressed in longitudinal direction and cannot describe phenomenon of loss of the pipeline stability. Therefore, when finding the value of critical force, they introduce a correction coefficient, which is set depending on incorrectly found calculated value of longitudinal force. Same researchers, when calculating SSS of a single-span above-ground beam crossing with compensators, do not use differential equation of longitudinal-transverse bending of the rod when it is stretched by longitudinal force, but the solution for equation of the rod bending, which does not depend on stop block of the compensators and internal pressure in the pipeline, under which the compensators stretch pipe in longitudinal direction during its bending.

Compiling the initial calculation data. In [10, 13, 14], a detailed description of compiling the initial calculation data for route profile and pipeline operation parameters is given. In the example we are considering the calculated underground section of oil pipeline in karst zone has the length of 300 m. Its middle part, 48 m long, was bared due to development of karst, i.e. there is no soil foundation and backfill soil. Sections of 24 m long adjacent to this middle part on the left and on the right are each in weakened soil. Under such operating conditions, there are single-span, uncompensated above-ground crossings without special supports, working in conjunction with adjacent underground sections of the pipeline [1, 8, 12]. During computer modeling of SSS for selected section of the oil pipeline, it is figuratively divided into 50 equal elementary parts, each of which has a length of 6 m. In accordance with longitudinal profile of the route for each of these parts, initial calculation data are selected: soil types indicating their bearing capacity; absolute marks; laying depth and height of backfill; radii and angles of rotation for bent branches; lengths of sections with a constant slope; pipe characteristics indicating the external diameter D_{ex} and wall thickness δ; category of pipeline section. According to execution documentation, the pipeline in calculated section is composed of pipes with $D_{ex} \times \delta - 1020 \times 20.0$ mm.

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Analysis of SSS for the pipeline section and assessment of internal pressure effect causing additional bending of the pipeline. Weight of the pipe with pumped product for the oil pipeline does not depend on internal pressure, which allows complete identifying the of ELAF S_x effect on bend of the pipeline. Calculation and analysis of SSS for calculated section of the pipeline is performed for two statements of the problem with different values of operating parameters. In the first statement, justification for which is given in this article, effect of internal pressure, which causes additional bending of the pipeline, is taken into account, and in the second statement this effect is neglected [4, 5]. Calculations were carried out for the following values of parameters considering the pipeline operation: 1) p_0 = 9.0 MPa; Δt = 29 °C; 2) p_0 = 4.5 MPa; Δt = 29 °C; 3) p_0 = 0.1 MPa; Δt = 29 °C; 4) p_0 = 9.0 MPa; Δt = 0 °C; 5) p_0 = 0.1 MPa; Δt = 0 °C, where Δt – temperature difference, which is equal to difference of temperatures in operation and pipeline closure during construction. Table shows extreme values for the main characteristics of the pipeline SSS for two statements of the problem: bend w; longitudinal displacement u; stresses from longitudinal forces σ_N; bending stresses σ_M; total longitudinal stresses $|\sigma^{s}_{\ln}|$ and their maximum permissible values $[\sigma^{s}_{\ln}]$ from specified impacts and loads [16] obtained from solving the problem about SSS for the oil pipeline at l_0 = 48 m, where l_0 denotes length of the part in the middle of calculated section and where due to active development of the karst there is no soil backfill, and soil foundation has exhausted bearing capacity and cannot counter longitudinal displacement of the pipe. In the parts that are from left and right adjacent to described above middle part, each 24 m long, soil foundation is also weakened due to development of karst, but it has not yet exhausted its bearing capacity, and natural balance has not yet been disturbed in backfill soil. In the first statement of the problem, effect of internal pressure in the pipeline, which causes additional pipe bending, i.e. according to the model, which was described in detail by us in the statement of the problem.

Results of calculating the maximum values of SSS characteristics of the oil pipeline

Calculations options		w, m	σ_N*, MPa	S_x, kN	σ_M**, MPa	$\|\sigma^{s}_{\ln}\|$, MPa	$[\sigma^{s}_{\ln}]$, MPa
The first statement of the problem	p_0 = 9.0 MPa; Δt = 29 °C	−0.54	0/12	7654/6828	219/–167	234	216
	p_0 = 4.5MPa; Δt = 29 °C	−0.49	–36/–27	6229/5580	190/–137	230	316
	p_0 = 0.1MPa; Δt = 29 °C	−0.44	–71/–64	4831/4308	168/–120	242	385
	p_0 = 9.0 MPa; Δt = 0 °C	−0.4	72/78	2845/2420	149/–106	230	385
	p_0 = 0.1 MPa; Δt = 0 °C	−0.35	5,6/1,1	7/–203	126/–84	129	385
The second statement of the problem	p_0 = 9.0 MPa; Δt = 29 °C	−0.35	0/3,8	58/–237	122/–86	127	385
	p_0 = 4.5MPa; Δt = 29 °C	−0.39	–30/–36	2427/2038	142/–101	181	316
	p_0 = 0.1MPa; Δt = 29 °C	−0.44	–71/–74	4747/4230	167/–120	241	386
	p_0 = 9.0 MPa; Δt = 0 °C	−0.29	71/75	–4701/–4979	97/–64	170	386
	p_0 = 0.1 MPa; Δt = 0 °C	−0.35	0.7/5.5	–71/–372	121/–86	129	386

Notes: * stresses at the ends of calculated section are shown in the numerator of the fraction, and in its middle part – in the denominator;
** stresses from span bending moment are given in the numerator of the fraction, and from the support – in the denominator.

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Of great interest is the analysis of solutions for the equations of theory of longitudinal and transverse bending of the pipeline, which are obtained from system of differential equations (2) in absence of joint deformations of the pipeline with soil and when ELAF S_x takes a constant value [1, 8, 12]. These equations are equivalent to well-studied differential equations of longitudinal-transverse bending of the rod in classical theory of rod bending [18]. Analysis showed that with an increase in internal pressure, ELAF S_x value increases, as well as extreme values of SSS characteristics of the pipeline.

In case when $p_0 = 9.0$ MPa; $\Delta t = 29$ °C, stresses value from longitudinal force $\sigma_N = 0$ MPa, therefore, magnitude of longitudinal force $N_x(x) = 0$ kN. If we take the viewpoint of some scientists [2, 4, 5], who believe that ELAF value determined by recommendations of regulatory document [16] is erroneous, i.e. if we do not consider the effect of internal pressure creating an additional bending of the pipeline in calculations, then in system of differential equations (2) it is necessary to neglect the term $\chi p_0 F_{\text{il}}$. Therefore, at indicated values of internal pressure and temperature, SSS characteristics of the pipeline should take lower values than corresponding SSS characteristics of the pipeline when internal pressure in the pipe decreases, since with a decrease in internal pressure the value of longitudinal force N_x increases, which stretches the pipe in longitudinal direction.

If we return to correct statement of the problem, then in fact this does not happen, because with an increase in internal pressure, ELAF S_x value also increases, due to which the pipeline experiences additional bending. For example, due to effect of ELAF S_x at $p_0 = 9.0$ MPa, $\Delta t = 29$ °C extreme value of bending increases by 54 %, extreme values of bending stresses from span bending moment increase by 74 %, and extreme-value of bending stresses from support bending moment doubles. With a decrease in internal pressure in the pipeline from $p_0 = 9.0$ to $p_0 = 4.5$ MPa same SSS characteristics of the pipeline only increase by 40, 67 and 39 %, respectively. Only due to effect of temperature stresses, i.e. at $p_0 = 0.1$ MPa, $\Delta t = 29$ °C indicated SSS characteristics of the pipeline increase by 26, 34 and 31 %, respectively. If there is no temperature difference, i.e. at $p_0 = 9.0$ MPa, $\Delta t = 0$ °C extreme value of bending increases by 14 %, extreme values of bending stresses from span bending moment increase by 32 %, and extreme value of bending stresses from support bending moment increases by 37 %.

In this regard, of great interest is calculating SSS of the pipeline section without taking into account the effect of internal pressure, which causes additional bending of the pipeline, i.e. without member $\chi p_0 F_{\text{il}}$ in system of differential equations (2). It is this approach that is used in some of above mentioned works, authors of which consider accounting a member $\left((\frac{1}{\rho_0}+\chi)\right)\left(-p_0F_{\text{il}}\right)$ as a significant mistake in this system. Therefore, in their opinion, in the system of differential equations (2), the term $\left((\frac{1}{\rho_0}+\chi)\right)\left(N_x-p_0F_{\text{il}}\right)$ must be replaced by $\left(\frac{1}{\rho_0}+\chi\right)\left(N_x\right)$. Some results of calculating the SSS of the pipeline with this correction are presented in a table, data analysis of which shows the following. At the ends of calculated section, where the pipeline is pinched by soil and is in a horizontal position at $p_0 = 9.0$ MPa; $\Delta t = 29$ °C we have $N_s = 0$ without considering $\chi p_0 F_{\text{il}}$ in a system of differential equations (2). Therefore, at a given ELAF S_x value, extreme SSS characteristics of the pipeline take minimum values compared to corresponding SSS characteristics of the pipeline with values of parameters for the pipeline operation presented in the table. For example, if internal pressure p_0 is reduced by half, i.e. from 9.0 to 4.5 MPa, extreme value of bend increases from 0.35 to 0.39 m (increases by more than 12 %). This is explained by the fact that as internal pressure decreases, value of longitudinal force N_x increases. In this case, system of differential equations (2) will be equivalent to differential equation of longitudinal-transverse bending of the rod when it is stretched by longitudinal force $N_x = 4747$ kN.

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Of practical interest is option of calculating SSS of the pipeline when $\Delta t = 0$ °C, i.e. there are no temperature stresses and the term $\chi p_0 F_{\text{il}}$ in the system of differential equations (2) is neglected. Analysis of data in the table shows that with an increase in internal pressure p_0 from 0.1 to 9.0 MPa extreme value of bend decreases by 17 %, extreme value of bending stresses from span bending moments – by 20 %, and extreme value of bending stresses from support bending moments – by 26 %. This result is explained by the fact that with an increase in internal pressure due to an incorrect determination of ELAF S_x, according to recommendations of [2, 4, 5], magnitude of this force begins to take a negative value, and system of differential equations (2) goes over to equation of longitudinal-transverse bending of the pipe when it is stretched in longitudinal direction [12, 18]. Moreover, error in calculating the extreme value of bend is 35 %, extreme value of bending stresses from span bending moments and extreme value of bending stresses from support bending moments are 44 % and 95 %, respectively. Error in calculating these characteristics at $p_0 = 9.0$ MPa; $\Delta t = 0$ °C, i.e. in absence of temperature stresses, for extreme value of bend is 28 %, and extreme value of bending stresses from span bending moments and extreme value of bending stresses from support bending moments is 35 % and 40 %, respectively. As expected in absence of internal pressure, causing additional bending of the pipe, i.e. at $p_0 = 0.1$ MPa; $\Delta t = 29$ °C or at $p_0 = 0.1$ MPa; $\Delta t = 0$ °C the results of calculating SSS characteristics of the pipeline for the first and second statements of the problems are identical.

Conclusion

1. To justify inclusion of internal pressure effect in the pipeline, causing additional bending of the pipe, in SSS mathematical model, and to refute the claims of some scientists [2, 4, 5] about erroneous introduction of ELAF S_x concept, determined depending on internal working pressure, temperature stresses and joint deformations of pipelines with various types of soils in [1, 11], as well as in regulatory document [16], analysis of the main differential equations describing SSS of the rod element is provided. When finding the displacements of nodal elements, displacement method is used, in which, in contrast to standard methods of finite differences and finite elements, resolving system of algebraic equations is composed at joint nodes of these elements taking into account peculiarities of the pipeline design, which allows reducing the main resolving system of algebraic equations by several orders of magnitude.

2. To justify accounting for internal pressure effect in the pipeline, causing additional bending of the pipeline, calculation and analysis of SSS for calculated section of the pipeline was performed for various values of operating parameters: taking into account effect of internal pressure, which causes its additional bending, and without taking it into account. It is shown that due to effect of ELAF S_x at $p_0 = 9.0$ MPa, $\Delta t = 29$ °C extreme value of bending increases by 54 %, extreme values of bending stresses from span bending moment increase by 74 %, and extreme value of bending stresses from support bending moment doubles. In case of neglecting the internal pressure effect causing additional bending of the pipeline (second statement of the problem), error in calculating the extreme value of bend is 35 %, extreme value of bending stresses from span bending moments is 44 %, and extreme value of bending stresses from support bending moments is 95 %.

3. According to some researchers [2, 4, 5], equation of longitudinal-transverse bending, magnitude of longitudinal force in [1, 6-8, 11, 12, 17, 19], as well as formulas for checking the longitudinal stability of the trunk pipelines, contained in regulatory document [16], have been composed with significant errors. If these conclusions are adopted, computer programs for calculating the durability and stability of the pipelines, as well as research results that take into account internal pres-

sure effect on bend of the pipeline in accordance with requirements of regulatory document [16], must be considered executed according to incorrect equations and erroneous formulas. They consider impossible the loss of stability for the pipeline with increasing internal pressure, whereas the pipe extends like a stretched string. Lack of analysis for solution of differential equation in these works led to incorrect results and incorrect conclusions. As a result, the conclusion contradicting common sense was made: to ensure durability of the pipeline, it is necessary to increase internal pressure in the pipe.

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